Virtualized Resource Sharing in Cloud Radio Access Networks Through Truthful Mechanisms

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***Abstract*— In the recent paradigm of cloud radio access networks (C-RAN), signal processing functions at the base stations (BSs) are virtualized and migrated into a mobile cloud that maintains a pool of virtual BS (VBS) instances. Remote radio heads and antennae at the BSs are connected to the VBS pool by fronthaul fiber links. Mobile operators may lease resources from the tower company who owns the C-RAN infrastructure. We study auction mechanisms for efficiently sharing C-RAN resources among mobile operators. Leveraging randomized rounding, we design an offline C-RAN auction mechanism that can achieve truthfulness and near-optimal social welfare. For the more realistic setting of online bid arrival, we design an online algorithm that executes in polynomial time and achieves a competitive ratio of $(1 - \epsilon)$. A tailored fractional Vickrey–Clarke–Groves mechanism works in concert with the online algorithm to elicit truthful bids. Extensive simulation studies verify the efficacy of our C-RAN auction mechanisms.**

***Index Terms*— C-RAN, network function virtualization, mechanism design.**

I. Introduction

The evolution of the current 4G and the upcoming 5G communication systems target user experience of 100 Mbps everywhere and 1 to 10 Gbps locally, with low latency compared to LTE. The mobile network infrastructure faces new challenges in managing large amounts of new spectrum, deploying new sites that enable new services and use cases [1]. In the current radio access network (RAN) architecture, base stations (BSs) are distributed and the processing capacity of a BS can be used only by its own users, and cannot be shared across BSs, not to mention across operators. Due to load imbalances, power efficiency at BSs is only 50%. Meanwhile, the cost for operators to construct and maintain BSs continues rising, while the utilization rate is as low as 1/3 [2].

It is therefore natural for mobile operators to consider sharing processing resources for cost reduction. The concept of Cloud Radio Access Network (C-RAN) recently arises from such context. Unlike the traditional RAN, C-RAN promises a higher degree of cooperation and communication among BSs [3], [4]. In C-RAN, Remote Radio Heads (RRHs) and antennas are located at the remote sites. Signal processing functions at BSs are virtualized and migrated into a central mobile cloud, which hosts a large pool of virtual base station (VBS) instances. Virtualization technology allows the implementation of VBSs as highly optimized virtual machine instances in the cloud datacenter. Low-latency high bandwidth optical fibers serve as fronthaul that interconnect RRHs and the VBS pool.

Based on centralization and virtualization of BS baseband processing, the benefits of C-RAN lie in the following aspects. First, centralization helps improve performance and reduce operational cost. The latter includes site rental, energy, support and maintenance expenses, and constitutes two thirds of the total network costs. With centralization, only the antenna is needed at the now greatly streamlined cell site. Operators in Korea, Japan and China have demonstrated between 30% to 50% reduction in operating expense (OPEX) [5]. Second, by applying network function virtualization (NFV) to the RAN, capital expenditures (CAPEX) such as equipment upgrade are significantly reduced. Using general-purpose servers for BS hardware enables effective sharing among operators. Furthermore, there is no need to replace the equipments when new standards or services are rolled out, thanks to NFV.

Since 2011, the telecom industry has started to migrate towards the C-RAN paradigm with pilot deployments and solutions. China Mobile cooperated with ZTE in their Changsha C-RAN trial [6]. Korea Telecom constructed the world's first commercial Cloud-RAN, Cloud Communications Center (CCC) using a virtualized network based on a Samsung/Intel platform [7]. However, the infrastructures proposed by China Mobile and Korea Telecom do not support inter-operator sharing. The telecom industry subsequently concretizes *the tower company*, who owns the C-RAN infrastructure, builds and maintains the BSs. In the US, American tower [8] is a global provider of wireless communication infrastructures including towers and antennas. In China, the national telecom tower company was founded in July 2014, with a plan to build 120,000 towers in three years [9]. Mobile operators can then focus on their telecom business, and rent resources from the tower company. The telecomm infrastructure, including BSs, can be flexibly shared and effectively utilized by operators.

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Given finite resource supply in a C-RAN system, when operators' demands exceed supply in the C-RAN market, the tower company needs an efficient mechanism to decide which operators to serve, and how to allocate resources in order to achieve the highest social welfare. Auction mechanisms represent a flexible and efficient approach towards such allocation of virtualized C-RAN resources. Different from simple allocation schemes based on fixed pricing, an auction is economically efficient, automatically discovers the realtime market value, and assigns resources to users who value them the most [10]. In a real-world C-RAN market, operator requests may arrive stochastically over time. Consequently, besides the offline mechanisms, it is also natural to consider online auction design. In the online setting, the auctioneer makes an irrevocable decision for each bid upon its arrival, regarding whether to serve that operator or not, without knowledge on future requests. Then it should dynamically assemble its finite spectrum and bandwidth resources to winning operators, and charge a carefully calculated payment to guarantee truthful bidding. It is worth noting that the social welfare maximization problem in the C-RAN auction is NP-hard in nature, hence efficient mechanism design is challenging even without the truthfulness constraint.

We focus on designing efficient auction mechanisms for C-RAN resource sharing. The tower company who owns the C-RAN infrastructure acts as the auctioneer and sells spectrum and bandwidth resources to the operators through an auction. In the offline setting, the auctioneer has knowledge of all the bids; while in the online setting, bids arrive sequentially, each to be accepted or rejected immediately. Our offline and online C-RAN auction design have the following goals. 1) *Truthfulness:* regardless of other operators' bids, each operator declares the true valuation which can maximize its utility. 2) *Computational efficiency:* The algorithms for winner decision and payment calculation run in polynomial time. 3) *Social welfare maximization:* We aim to maximize the social welfare on aggregate utilities of the tower company and the operators.

As a first step of the C-RAN market mechanism design, we formulate the social welfare maximization problem as an integer linear program (ILP), which is proven NP-hard. Thus, instead of directly computing the optimum solution, we resort to a fast approximation algorithm based on randomized rounding, to compute a near-optimal solution in expected polynomial time. Auction design in recent networking literature have been heavily relying on the primal-dual framework, some of these require a sampling phase before allocating the resources and need to know the capacity ratio upfront [11]. The capacity ratio is a ratio between minimum resource capacity and maximum bidder's resource request which is used to measure how many bidders might win. We depart from this primal-dual approach and resort to the classic primal-only method based on randomized rounding, which requires no sampling phase and needs very little C-RAN system information.

We first design a near-optimal approximation algorithm to solve the offline C-RAN resource allocation problem. Then in the more challenging online setting, the operators' bids come one at a time and have to be decided to win or not immediately without knowledge of future bids. We study the online auction in the random order model, where the arrival order of the bids is chosen uniformly at random out of all possible permutations. The winner determination process in our online mechanism depends on the known bidding prices and resource demands, and then predicts future bids' influence by scaling down the resource capacity with a factor f in deciding the tentative winners. The tentative allocation is carried out if the total spectrum and bandwidth resource demand does not exceed the corresponding capacity. Our online algorithm is carefully tuned to achieve a $(1 - \epsilon)$ competitive ratio, defined as the ratio between the expected social welfare produced by the online algorithm and the offline optimal social welfare, where $(1 - \epsilon)$ is around 0.8 in practical settings. To guarantee truthful bidding from mobile operators who may be strategic in practice, it is necessary to design an accompanying payment mechanism for eliciting truthful bids. The classic Vickrey-Clarke-Groves (VCG) mechanism while known to be truthful, requires optimal solution to the social welfare maximization problem, which is proven NP-hard. We instead adapt a version of the fractional VCG mechanism for computing the payments, to cooperate with the online allocation algorithm.

In the rest of the paper, we review the previous literature in Sec. II. The C-RAN auction system model is discussed in Sec. III. The offline and online C-RAN auctions are presented in Sec. IV. and Sec. V, respectively, and are evaluated in Sec. VI. Sec. VII concludes the paper.

II. Related Work

Along the direction of C-RAN architecture and operations, a series of recent studies propose to implement new platforms to improve the traditional RAN. Zhu *et al.* [12] present the first working prototype of a virtual BS pool as a step to realize wireless network cloud and show the feasibility of MAC and PHY using software radio. Pompili *et al.* [3] present reactive and proactive resource provisioning and allocation strategies of Virtual BSs in the centralized resource pool in C-RAN to improve resource utilization efficiency and system performance. Resource sharing in Heterogeneous Cloud Radio Access Networks (H-CRAN) [13], [14] which incorporates HetNet and C-RAN is studied to improve spectral efficiency, reduce CAPEX and OPEX. Yang *et al.* [15] propose Open-RAN, an architecture for software-defined RAN via virtualization, towards the goal of making the traditional RAN more open, controllable, flexible and evolvable. Pompili *et al.* [16] study elastic VBSs and dynamic RRH density to accommodate the fluctuations in capacity demand through demand-aware dynamic VM provisioning and allocation. However, those works only focus on resource allocation in virtual base station pool without considering the role of fiber links and RRHs.

As is known to all, C-RAN consists of three main components: Virtual BS pool, fiber links and RRHs. Besides computing resources allocation and management problem in VBS pool, the main design challenge for the fiber links and RRHs is the link and RRH selection problem [17]. Tang *et al.* [18] propose a cross-layer resource allocation model to optimize the set of selected RRHs and the beamforming strategies at the active RRHs for purpose of minimizing the overall system power consumption. Moreover, [19] considers joint RRH selection and fronthaul beamforming to minimize the system power consumption, and [20] considers joint BS selection and distributed compression in C-RAN to improve the energy efficiency. In these existing works, C-RAN infrastructure owner (*e.g.* tower company) directly allocates the physical resources to mobile users without involving the operators. Actually, tower campany is only responsible for sell resources to each operator, while each operator allocates those resources to its users.

Therefore, we design auction mechanisms to address the RRHs and fiber links resource allocation problem in C-RAN. Auctions have long been adopted as an efficient mechanism for allocating physical and virtual resources to competing users. Zhang *et al.* [21] focus on dynamic cloud resource provisioning, and design a randomized auction for VM transactions in the cloud market. Shi *et al.* [22] design an intuition-driven primal-dual algorithm for translating the online social welfare optimization problem into a series of one-round optimization. Zhang *et al.* [23] design truthful and efficient online VM auctions that target social welfare and profit maximization with server costs in IaaS Clouds. Auction approaches are also widely used to solve resource allocation problems in wireless networks [24]–[26]. Zhou *et al.* [27] propose a truthful and computationally-efficient spectrum auction to improve the spectrum utilization for wireless networks. Wei *et al.* [28] study SHIELD, a truthful auction mechanism for channel allocation in multi-radio, multi-channel non-cooperative wireless networks, which guarantees both strategy-proofness and high system performance. Zhu and Hossain [29] propose a hierarchical combinatorial auction mechanism to jointly address the two-level resource allocation problem in 5G cellular networks. However, in the existing wireless auction mechanisms, they only consider the situation that all the requests are known in advance (*i.e.* offline scenario). In real-world C-RAN market, operators probably arrive one by one to buy resources from the tower company and are not willing to wait for other bidders' arrival. With online auction, the tower company can serve each bidder immediately without knowledge of future. Hence, it is natural to consider the problem in online scenario.

Departing from the recently popular primal-dual framework, Kesselheim *et al.* [11] study packing LPs in an online setting using random rounding, towards designing highly-competitive online algorithms. Archer *et al.* [30] modify the randomized rounding algorithm for combinatorial auctions with single parameter agents to make it monotone, and derive an approximately efficient truthful mechanism. This work has been partly inspired by the recent renaissance of primal-only techniques, and targets both offline and online auctions to allocate resources in fiber links and RRHs and to maximize social welfare in a C-RAN system.

III. System Model

A. Our C-RAN Auction System

We consider a Cloud Radio Access Network (C-RAN) market where M base stations are connected to the pool of Virtual BSs by I optical fibers, as shown in Fig. 1. Our model supports two types connection strategy: *point to point*, *i.e.*, every BS has its own fiber to the cloud, and *daisy chain*, *i.e.*, BSs share one fiber link to the cloud. Thus, the total number of fibers is no more than the total number of base stations, *i.e.*, $I \leq M$. Each fiber link $i \in [I]$ has a bandwidth capacity that is denoted by D_i. The total amount of spectrum resources for BS $m \in [M]$ is denoted by R_m. Those Virtual BS instances are allocated in a cloud data center which contains sufficient computing resources including CPU, RAM and storage.

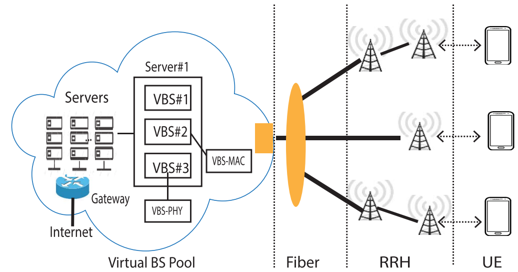

Fig. 1. An illustration of the C-RAN auction model.

The auctioneer who owns the C-RAN infrastructure sells two kinds of resources: spectrum resources at BSs and bandwidth at optical fiber links. A mobile operator who acts as the bidder submits a bid to compete for the limited spectrum resources and link bandwidth. During the online auction, when one bid arrives, we make a decision whether to serve it or not immediately, aiming to pursue the maximum social welfare. Suppose there are in total N bidders, each submitting one bid with a set of K options. Each option specifies a price the operator is willing to pay, for a certain amount of resource consumption. That is, bidder $n \in [N]$'s option $k \in [K]$ corresponds to the variable $x_{n,k}$ raises the price $b_{n,k}$ while having spectrum resources consumption $r_{n,k,m}$ for every Base Station m and bandwidth consumption $d_{n,k,i}$ for every link i. Assume both $r_{n,k,m}$ and $d_{n,k,i}$ are constants no smaller than 1. Adopting the XOR bidding rule [21], we assume that only a single option of bidder n can be selected.

Our auction design targets the following goals: 1) *individual rationality*: Each bidder's utility u_n is non-negative, *i.e.*, $u_n \geq 0, \forall n \in [N]$. If bidder n wins, we define the utility function as $u_n = \sum_{k \in [K]} v_{n,k} x_{n,k} - p_n$, where $v_{n,k}$ is the true valuation of its option k and p_n is the payment. 2) *Truthfulness*: For each operator in our C-RAN auction, bidding the true valuation for each option k can always maximizes an operator's utility, regardless of other operators' bids. Thus, if the auction is truthful, we can safely assume that $v_{n,k} = b_{n,k}, \forall n \in [N], \forall k \in [K]$. 3) *Social welfare maximization*: in the C-RAN ecosystem, the total social welfare is the sum of the cloud owner's revenue and the operators' utilities, *i.e.*, $\sum_{n \in [N]} u_n + \sum_{n \in [N]} p_n = \sum_{n \in [N]} \sum_{k \in [K]} b_{n,k} x_{n,k}$.

Under truthful bidding, we can formulate the winner determination problem for social welfare maximization into the following integer linear program (ILP):

$$\text{Maximize} \sum_{n \in [N]} \sum_{k \in [K]} b_{n,k} x_{n,k} \tag{1}$$

Subject to:

$$\sum_{n\in[N]}\sum_{k\in[K]} r_{n,k,m}x_{n,k} \le R_m, \quad \forall m \in [M], \tag{1a}$$

$$\sum_{n\in[N]}\sum_{k\in[K]} d_{n,k,i}x_{n,k} \le D_i, \quad \forall i \in [I], \tag{1b}$$

$$\sum_{k\in[K]} x_{n,k} \le 1, \quad \forall n \in [N], \tag{1c}$$

$$x_{n,k} \in \{0,1\}, \quad \forall n \in [N],\ k \in [K]. \tag{1d}$$

Constraint (1a) indicates that the total spectrum resource requests from winning bids can not exceed the capacity of each BS. Constraint (1b) guarantees that the total data rate is bounded by the capacity of each fiber link's bandwidth. Constraint (1c) restricts that only one of the K options from each bidder can be selected. With Constraint (1d), we know that $x_{n,k}$ is a binary variable denoting whether bidder n's k-th option is accepted or not. Our goal is to maximize the social welfare. However, ILP (1) with only one of the constraint (1a) is a 0-1 Knapsack problem. We can say that because the classical NP-hard knapsack problem is a special case of our problem, the C-RAN auction is also NP-hard. We therefore resort to efficient approximate solution.

We consider the LP relaxation of ILP (1) by relaxing (1d) to $0 \le x_{n,k} \le 1, \forall n \in [N], k \in [K]$. We use OPT^f to represent the optimal fractional objective function value of the LP relaxation and ALG to denote the integral objective value of the ILP (1) computed by the algorithm. In reality, the operator may only bid for resources from a subset of BSs and fiber links rather than all of them. We assume that every mobile operator requests spectrum resources from at most h_1 BSs and link bandwidth from at most h_2 fiber links. Thus, by definition, we know that $1 \le h_1 < M$ and $1 \le h_2 < I$. We define $\max_{n\in[N],k\in[K]} r_{n,k,m} = r_{\max}$ and $\max_{n\in[N],k\in[K]} d_{n,k,i} = d_{\max}$ where $r_{\max}$ and $d_{\max}$ are two constant numbers that indicate respectively the max atomic spectrum and bandwidth demand. Let $R_{\min}$ and $D_{\min}$ denote $\min_{m\in[M]} R_m$ and $\min_{i\in[I]} D_i$, respectively. We can now define the capacity ratio for the spectrum resource as $C_1 = \min_{m\in[M]} \frac{R_m}{\max_{n\in[N],k\in[K]} r_{n,k,m}} = \frac{R_{\min}}{r_{\max}}$, and the capacity ratio for bandwidth as $C_2 = \min_{i\in[I]} \frac{D_i}{\max_{n\in[N],k\in[K]} d_{n,k,i}} = \frac{D_{\min}}{d_{\max}}$. Let $h = h_1 + h_2$ denote the maximum total number of BSs and links an operator can bid at, $C = \min\{C_1, C_2\}$ denote the capacity ratio for the overall problem, and $\Gamma = \min\{R_{\min}, D_{\min}\}$ is the smallest resource capacity. Table I summarizes key notations in the paper.

B. The Online Auction Problem

In our online C-RAN auction, bidders come one by one in a random order. The auctioneer who owns the C-RAN infrastructure needs to decide whether each bidder wins or not immediately based on hitherto information available, with potential future bids kept in mind. We consider a scaled LP in each round l with a scaling factor $f = \frac{l}{N}$ and the current set of arrived bidders $S \subset [N]$. Let $\mathcal{P}(f, S)$ denote the set of feasible solutions of the scaled LP in which every R_m and D_i are scaled by a factor f and only bidders in set S are served.

TABLE I
SUMMARY OF NOTATION

N	# of bidders	I	# of fibre links
M	# of base stations	S	the current subset of bidders
p_n	bidder n's payment	C_1	capacity ratio for spectrum resource
u_n	bidder n's utility	C_2	capacity ratio for link bandwidth
f	the scaling factor	D_i	capacity of ith fiber link
K	# of options that each bidder provides		
$b_{n,k}$	bidding price of bidder n's option k		
$v_{n,k}$	true valuation of bidder n's option k		
$x_{n,k}$	bidder n's option k is selected (1) otherwise (0)		
$r_{n,k,m}$	spectrum resources at BS m of bidder n's option k		
$r_{\max}$	the max spectrum demand from a single BS		
R_m	the total amount of spectrum resources at BS m		
$R_{\min}$	the minimum BS spectrum capacity		
$d_{n,k,i}$	bandwidth at link i requested by bidder n's option k		
$d_{\max}$	the max bandwidth demand from a single link		
$D_{\min}$	the minimum fiber link bandwidth capacity		
Γ	$\Gamma = \min\{R_{\min}, D_{\min}\}$,the smallest resource capacity		
ALG	objective value of the ILP (1) computed by the algorithm		
OPT	optimal objective value to the ILP (1)		
OPT^f	fractional optimum to the LP relaxation of ILP (1)		
C	$C = \min\{C_1, C_2\}$ capacity ratio for the whole system		
h_1	max # of BSs that each bidder bids spectrum from		
h_2	max # of links that each bidder bids bandwidth from		
h	$h = h_1 + h_2$, the max total # of bidding BS and links		
$\mathcal{P}(f, S)$	set of feasible solutions of scaled LP (2)		

Algorithm 1 Offline Algorithm for ILP (1)

```
Solve the LP relaxation of ILP (1), obtain the vector x̃
for each bidder n do
    Choose an option k with probability (1 − ε′/2)x̃_{n,k};
    Set the corresponding variable x_{n,k} = 1, set to 0
        otherwise
end for
Repeat the above for loop until it produced a feasible
    solution with social welfare at least (1 − ε′)OPT^f
```

The scaled LP is as follows:

$$\text{Maximize} \sum_{n\in[N]}\sum_{k\in[K]} b_{n,k}x_{n,k} \tag{2}$$

Subject to:

$$\sum_{n\in[N]}\sum_{k\in[K]} r_{n,k,m}x_{n,k} \le fR_m, \quad \forall m \in [M], \tag{2a}$$

$$\sum_{n\in[N]}\sum_{k\in[K]} d_{n,k,i}x_{n,k} \le fD_i, \quad \forall i \in [I], \tag{2b}$$

$$0 \le \sum_{k\in[K]} x_{n,k} \le 1, \quad \forall n \in S,$$

$$x_{n,k} = 0, \quad \forall n \notin S \tag{2c}$$

IV. The Offline C-RAN Auction Mechanism

We first consider the offline scenario where bids are all given at once. We consider the LP relaxation for ILP (1), and design the offline resource allocation algorithm (Algorithm 1) for the C-RAN auction. After that, pricing schemes are designed to ensure truthfulness.

In Algorithm 1, we first solve the LP relaxation of ILP (1) for an optimal fractional allocation $\tilde{x}$ in line 1. The next step is to round $\tilde{x}$ to an integer solution x. Employing randomized rounding techniques, we make an independent random selection for each bidder n with proper probabilities. Because randomized rounding can lead to violation of a constraint, towards increasing feasibility of the temporary allocation result x, the solution is to scale $\tilde{x}$ with a fraction smaller than 1 as the probabilities. We assign bidder n the option k with probability $(1-\frac{\epsilon'}{2})\tilde{x}_{n,k}$, where $\epsilon' \in (0,1)$ is a small number, instead of directly using $\tilde{x}_{n,k}$ as probabilities. In Algorithm 1, the `for` loop in lines 2-5 chooses an option k with probability $(1-\frac{\epsilon'}{2})\tilde{x}_{n,k}$ for each bidder n. Line 6 tunes the temporary solution x feasible to the C-RAN system. We next examine the approximation ratio and running time of our offline algorithm. We first bound the probability that one of the constraints in ILP (1) is infeasible under the temporary allocation solution x.

Theorem 1 (Chernoff Bound [31]): Let $X_1, \ldots, X_N$ be independent Poisson trials such that, for $1 \le n \le N$, $\Pr[X_n = 1] = p_n$, where $0 < p_n < 1$. Then, for $X = \sum_{n=1}^{N} X_n$, $\mu \ge \mathbf{E}[X] = \sum_{n=1}^{N} p_n$, and any $0 < \alpha < 2e-1$, we have $\Pr[X > (1+\alpha)\mu] < e^{-\mu\alpha^2/4}$

Lemma 1: In our C-RAN auction system, let $c > 0$, the capacity ratio $C = \Omega(\frac{\ln h}{\epsilon'^2})$ (the constant inside Ω is $16(c+1)$). Let Φ denote the event that a given BS's spectrum or a given fiber link's bandwidth is over-sold. Then the probability that event Φ happens is at most $\frac{1}{h^{c+1}}$.

Proof: Recall that C is defined as $\min\limits_{m\in[M], i\in[I]}\{\frac{R_m}{r_{\max}}, \frac{D_i}{d_{\max}}\}$ and Φ is the event that one of the constraints in (1a) or (1b) is violated. Let Φ_1 and Φ_2 be the events that one of the constraints in (1a) and (1b) is violated, respectively. We have $\Pr[\Phi_1] = \Pr[\sum_{n\in[N]}\sum_{k\in[K]} r_{n,k,m}x_{n,k} > R_m] \le \Pr[\sum_{n\in[N]}\sum_{k\in[K]} r_{\max}x_{n,k} > R_m] = \Pr[\sum_{n\in[N]}\sum_{k\in[K]} x_{n,k} > \frac{R_m}{r_{\max}}]$. Similarly, we have $\Pr[\Phi_2] = \Pr[\sum_{n\in[N]}\sum_{k\in[K]} d_{n,k,i}x_{n,k} > D_i] \le \Pr[\sum_{n\in[N]}\sum_{k\in[K]} x_{n,k} > \frac{D_i}{d_{\max}}]$. Therefore, $\Pr[\Phi] = \Pr[\Phi_1 \cup \Phi_2] \le \Pr[\sum\limits_{n\in[N]}\sum\limits_{k\in[K]} x_{n,k} > \min\limits_{m\in[M], i\in[I]}\{\frac{R_m}{r_{\max}}, \frac{D_i}{d_{\max}}\}] = \Pr[X > C]$, where $X = \sum_{n\in[N]}\sum_{k\in[K]} x_{n,k}$. Instead of constraint (1a) and (1b), we consider the following constraint $\sum_{n\in[N]}\sum_{k\in[K]} x_{n,k} \le C$ and the new LP (3a). Then, we can bound $\Pr[X > C]$ using *Theorem 1*. Run Algorithm 1 on the new LP (3a),

$$\text{Maximize} \quad \sum_{n\in[N]}\sum_{k\in[K]} b_{n,k}x_{n,k} \tag{3}$$

$$\text{Subject to:} \quad \sum_{n\in[N]}\sum_{k\in[K]} x_{n,k} \le C, \tag{3a}$$

$$\sum_{k\in[K]} x_{n,k} \le 1, \quad \forall n \in [N] \tag{3b}$$

Let $\widehat{x}$ denote the optimal solution to the LP (3a) from line 1 and x' is a temporary integer solution to LP (3a) obtained from line 4. From Algorithm 1, we know $\Pr[x'_{n,k} = 1] = (1-\frac{\epsilon'}{2})\widehat{x}_{n,k}$. We define $X'_n = \sum_{k\in[K]} x'_{n,k}$, $X' = \sum_{n\in[N]} X'_n$. We can apply a union bound on X'_n to get $\Pr[X'_n = 1] \le \sum_{k\in[K]}\Pr[x'_{n,k} = 1] = (1-\frac{\epsilon'}{2})\sum_{k\in[K]}\widehat{x}_{n,k}$. Therefore, $\mathbf{E}[X'] = \sum_{n\in[N]}\Pr[X'_n = 1] \le (1-\frac{\epsilon'}{2})C$. Applying *Theorem 1*, because $C \ge \frac{16(c+1)}{\epsilon'^2}\ln h$, $\mu = (1-\frac{\epsilon'}{2})C \ge \mathbf{E}[X']$ and $\alpha = \frac{\frac{\epsilon'}{2}}{1-\frac{\epsilon'}{2}} < 2e-1$, we have:

$$\begin{aligned}\Pr[X' > C] &\le \exp\big(-(1-\frac{\epsilon'}{2})C(\frac{\frac{\epsilon'}{2}}{1-\frac{\epsilon'}{2}})^2/4\big)\\ &\le \exp\big(-\frac{c+1}{1-\frac{\epsilon'}{2}}\ln h\big)\\ &= h^{-\frac{c+1}{1-\frac{\epsilon'}{2}}} \le \frac{1}{h^{c+1}}\end{aligned}$$

Thus, we obtain $\Pr[\Phi] \le \Pr[X' > C] \le \frac{1}{h^{c+1}}$. □

Theorem 2: if $C = \Omega(\frac{\ln h}{\epsilon'^2})$, Algorithm 1 can compute a feasible allocation with social welfare at least $(1-\epsilon')\text{OPT}^f$ in expected polynomial running time.

Proof: We can see from Lemma 1 that with high probability, no BS spectrum or fiber link bandwidth is over-sold. Taking a Union Bound on at most h BSs and links, we find that the temporary integer solution x from the `for` loop in lines 2-5 is feasible with probability at least $1-\frac{1}{h^c}$. The expected social welfare of the temporary integer solution is

$$\begin{aligned}\mathbf{E}[b^T x] &= \sum_{n\in[N]}\sum_{k\in[K]} b_{n,k}\mathbf{E}[x_{n,k}]\\ &= \sum_{n\in[N]}\sum_{k\in[K]} b_{n,k}(1-\frac{\epsilon'}{2})\tilde{x}_{n,k}\\ &= (1-\frac{\epsilon'}{2})\text{OPT}^f\end{aligned}$$

Algorithm 1 computes a satisfactory allocation that is feasible and has social welfare at least $(1-\epsilon')\text{OPT}^f$ with a number of independent trials in line 6. If the probability of obtaining a satisfactory allocation is p, the number of independent trials needed before finding the satisfactory allocation is a geometric random variable with expectation $\frac{1}{p}$. Hence, if $\frac{1}{p} = O(1)$, we can say that Algorithm 1 runs in expected polynomial time. Infeasible solutions occur with a probability $\frac{1}{h^c}$ and contribute at most $n \times \text{OPT}^f$ when every bidder wins its favourite bid. We assume $n \ll h^c$, infeasible solutions contribute essentially nothing to the expected social welfare of the algorithm. Because feasible solutions contribute at most OPT^f, applying averaging argument [32], we have $\mathbf{E}[b^T x] = \sum\limits_{w<(1-\epsilon')\text{OPT}^f} w\Pr[b^T x = w] + \sum\limits_{w\ge(1-\epsilon')\text{OPT}^f} w\Pr[b^T x = w] \le (1-\epsilon')\text{OPT}^f(1-p) + \text{OPT}^f p$. Because $\mathbf{E}[b^T x] = (1-\frac{\epsilon'}{2})\text{OPT}^f$, we can get $(1-\frac{\epsilon'}{2})\text{OPT}^f \le (1-\epsilon')\text{OPT}^f(1-p) + \text{OPT}^f p$. Therefore, we can bound the probability $p \ge \frac{1}{2}$ and the expected number of iterations for Line 6 is $\frac{1}{p} = O(1)$. In Line 1, we can solve the LP in polynomial time using the interior point method. The `for` loop in Lines 2-5 runs in $O(KN)$ time. In summary, the expected running time of Algorithm 1 is polynomial and it returns a $(1-\epsilon')$-approximate social welfare. □

The previous analyses are all based on the assumption that all mobile operators bid with their true valuations. To guarantee truthfulness, we need to design a payment strategy for each winning bidder. The classic

VCG (Vickrey-Clarke-Groves) [33] mechanism is a natural candidate. In VCG auctions, bidders pay their social cost, *i.e.*, the externality that it exerts on the other bidders. Because our C-RAN auction problem is NP-hard, the proposed offline algorithm 1 can only compute an approximate solution to ILP (1). The VCG mechanism, however, requires optimal solutions to the social welfare maximization problem or allocations that satisfy *maximal-in-distributional-range* (MIDR) rule [31]. We will next use the decomposition-based mechanism proposed by Lavi and Swamy [34] to convert the $(1-\epsilon')$-approximate randomized rounding algorithm (Algorithm 1) into a $(1-\epsilon')$-approximate allocation rule that is MIDR.

Definition 1 (The Decomposition-Based Mechanism [34]):

1. Compute an optimal fractional solution $\tilde{x}$ and VCG payment p_n^F.
2. Obtain a decomposition $(1-\epsilon')\tilde{x} = \sum_{\omega\in\Omega} \lambda_\omega x^\omega$.
3. With probability λ_ω: Choose allocation x^ω and set payments $p_n^I = \frac{\sum_{k\in[K]} b_{n,k} x_{n,k}^\omega}{\sum_{k\in[K]} b_{n,k}\tilde{x}_{n,k}} p_n^F$.

Using the decomposition-based mechanism, we first compute the fractional optimum $\tilde{x}$ and fractional VCG payment $p_n^F = \max\limits_{x\in\mathcal{P}(1,[N]\setminus\{n\})} b^T x - \sum\limits_{n'\in[N]\setminus\{n\}} \sum\limits_{k\in[K]} b_{n',k}\tilde{x}_{n',k}$ in polynomial time. We define $E = \{(n,k)|\tilde{x}_{n,k} > 0\}$ that has polynomial size because $\tilde{x}$ can be computed in polynomial time. Then in the second step, let $\{x^\omega\}_{\omega\in\Omega}$ denote all integral solutions. We need to find the probability distribution $\{\lambda^\omega\}_{\omega\in\Omega}$ such that for all $\omega \in \Omega$, $\lambda^\omega \geq 0$, $\sum_{\omega\in\Omega}\lambda^\omega = 1$ and $\sum_{\omega\in\Omega}\lambda_\omega x_{n,k}^\omega = (1-\epsilon')\tilde{x}_{n,k}$. Although Ω contains exponentially many variables, we show that the second step can still be solved in polynomial time. In the last step, we choose an allocation x^ω with probability λ^ω and compute the payment p_n^I which can guarantee truthfulness in expectation. We formulate the decomposition problem into a pair of primal and dual linear programs P and D:

$$
\begin{aligned}
&\text{Minimize} \sum_{\omega\in\Omega}\lambda_\omega \quad (P)\\
&\text{s.t.:} \sum_{\omega\in\Omega}\lambda_\omega x_{n,k}^\omega = (1-\epsilon')\tilde{x}_{n,k}, \quad \forall (n,k)\in E\\
&\qquad \sum_{\omega\in\Omega}\lambda_\omega \geq 1\\
&\qquad \lambda_\omega \geq 0, \quad \forall \omega\in\Omega\\
&\text{Maximize } (1-\epsilon') \sum_{(n,k)\in E} \tilde{x}_{n,k} z_{n,k} + Z^* \quad (D)\\
&\text{s.t.:} \sum_{(n,k)\in E} x_{n,k}^\omega z_{n,k} + Z^* \leq 1, \quad \forall\omega\in\Omega\\
&\qquad Z^* \geq 0\\
&\qquad z_{n,k} \text{ unrestricted}, \quad \forall (n,k)\in E
\end{aligned}
$$

Lemma 2: The optimum of the primal and dual linear program is 1, and the decomposition $(1-\epsilon')\tilde{x} = \sum_{\omega\in\Omega}\lambda_\omega x^\omega$ can be computed in polynomial time.

Proof: We first show that the dual optimum is 1. Suppose $(1-\epsilon')\sum_{(n,k)\in E}\tilde{x}_{n,k}z_{n,k} + Z^* > 1$. Let Algorithm 1 receive z as input and output the integral allocation x^ω. Because Algorithm 1 is $(1-\epsilon')$-approximate, we have $\sum_{(n,k)\in E} x_{n,k}^\omega z_{n,k} \geq (1-\epsilon')\sum_{(n,k)\in E}\tilde{x}_{n,k}z_{n,k} > 1 - Z^*$. The second inequality is by assumption. Thus, the first constraint in the dual LP is violated for x^ω. It indicates that the optimum is at most 1. Furthermore, a feasible solution to the dual program is given by $Z^* = 1$ and $z_{n,k} = 0$ for all $(n,k) \in E$, which indicates the optimum is at least 1. Thus, the dual optimum is exactly 1. Because of *strong duality*, the primal optimum is also 1.

The preceding discussion shows that the dual program D can be augmented with a redundant inequality $(1-\epsilon')\sum_{(n,k)\in E}\tilde{x}_{n,k}z_{n,k} + Z^* \geq 1$. Recall that the primal program P has polynomial constraints but exponentially many variables. Thus, its dual program D has polynomially many variables but an exponential number of constraints. We can run the ellipsoid method [35] on the modified dual program to reduce it to an equivalent dual program D' with polynomially many constraints. Those constraints are the violated inequalities returned by a *separation oracle*, constructed from Algorithm 1 as follows: if $(1-\epsilon')\sum_{(n,k)\in E}\tilde{x}_{n,k}z_{n,k}+Z^* > 1$, we can use Algorithm 1 to find a violated inequality. Otherwise, we can simply use the half space $(1-\epsilon')\sum_{(n,k)\in E}\tilde{x}_{n,k}z_{n,k} + Z^* \geq 1$. Therefore, D''s corresponding primal program P' has a polynomial number of variables and constraints, and can be solved in polynomial time. □

Theorem 3: Assume the C-RAN auction system has capacity ratio $\Omega(\frac{\ln h}{\epsilon'^2})$. Our C-RAN auction payment mechanism guarantees truthfulness in expectation and approximates optimal social welfare with a ratio $1-\epsilon'$.

Proof: In our C-RAN auction, bidder n comes with K options and bids $b_{n,k}$ for each option $k \in [K]$. The utility function u_n^F in the fractional VCG auction is defined as the difference between its valuation and the payment, *i.e.*, $u_n^F = \sum_{k\in[K]} b_{n,k}\tilde{x}_{n,k} - p_n^F$. We first prove that the fractional VCG payment is individually rational, *i.e.*, $u_n^F \geq 0$

$$
\begin{aligned}
u_n^F &= \sum_{k\in[K]} b_{n,k}\tilde{x}_{n,k} - p_n^F\\
&= \sum_{k\in[K]} b_{n,k}\tilde{x}_{n,k} - \max_{x\in\mathcal{P}(1,[N]\setminus\{n\})} b^T x\\
&\quad + \sum_{n'\in[N]\setminus\{n\}}\sum_{k\in[K]} b_{n',k}\tilde{x}_{n',k}\\
&= \max_{x\in\mathcal{P}(1,[N])} b^T x - \max_{x\in\mathcal{P}(1,[N]\setminus\{n\})} b^T x
\end{aligned}
$$

$\max_{x\in\mathcal{P}(1,[N])} b^T x$ is the max social welfare with total N bidders, while $\max_{x\in\mathcal{P}(1,[N]\setminus\{n\})} b^T x$ is the max social welfare with the same amount of resources and one less bidder n. Thus, $u_n^F \geq 0$.

Now we assume bidder n lies and reports a false value $\hat{b}_{n,k}$ for its option k. Then the fractional allocation decision based on the false value will become $\hat{x}$. The utility under the false bid is $\hat{u}_n^F = \sum_{n\in[N]}\sum_{k\in[K]} b_{n,k}\hat{x}_{n,k} - \max_{x\in\mathcal{P}(1,[N]\setminus\{n\})} b^T x$. The difference between the two utilities is:

$$
u_n^F - \hat{u}_n^F = \max_{x\in\mathcal{P}(1,[N])} b^T x - \sum_{n\in[N]}\sum_{k\in[K]} b_{n,k}\hat{x}_{n,k}
$$

Because $\max_{x\in\mathcal{P}(1,[N])} b^T x$ is the maximum social welfare, $\max_{x\in\mathcal{P}(1,[N])} b^T x \geq \sum_{n\in[N]}\sum_{k\in[K]} b_{n,k}\hat{x}_{n,k}$. Thus, $u_n^F - \hat{u}_n^F \geq 0$. We can say that under the fractional VCG auction,

Algorithm 2 One-Round Randomized Algorithm A_{round}

Input: $S, l, N, \{R_m\}, \{D_i\}, \{b_{n,k}, r_{n,k,m}, d_{n,k,i}\}, \forall n \in S$

Output: fractional optimum $\tilde{x}^{(l)}$ to LP (2), integral solution $x^{(l)}$ for round l

1: Let $\tilde{x}^{(l)}$ be an optimal solution to the scaled LP $\max_{x \in \mathcal{P}(\frac{l}{N}, S)} b^T x$;

2: Choose an option $k^{(l)}$ (possibly none) where option k has probability $\tilde{x}^{(l)}_{n,k}$;

3: Define $x^{(l)}$ with $x^{(l)}_{n',k} = \begin{cases} 1, \text{if } n' = n \text{ and } k = k^{(l)}; \\ 0, \text{otherwise}; \end{cases}$

return $\tilde{x}^{(l)}$, $x^{(l)}$

a mobile operator's utility can not be increased by a falsified bid. Thus, the fractional VCG auction is truthful.

Then an allocation x^{ω} is chosen with probability λ_{ω}. Each winning bidder n is charged with payment p^I_n. We calculate the expectation of bidder n's utility:

$$\begin{aligned} U^I_n &= \sum_{\omega \in \Omega} \lambda_{\omega} (\sum_{k \in [K]} b_{n,k} x^{\omega}_{n,k} - p^I_n) \\ &= \frac{\sum_{k \in [K]} b_{n,k} \sum_{\omega \in \Omega} \lambda_{\omega} x^{\omega}_{n,k}}{\sum_{k \in [K]} b_{n,k} \tilde{x}_{n,k}} (\sum_{k \in [K]} b_{n,k} \tilde{x}_{n,k} - p^F_n) \\ &= \frac{\sum_{k \in [K]} b_{n,k} (1-\epsilon') \tilde{x}_{n,k}}{\sum_{k \in [K]} b_{n,k} \tilde{x}_{n,k}} (\sum_{k \in [K]} b_{n,k} \tilde{x}_{n,k} - p^F_n) \\ &= (1-\epsilon') (\sum_{k \in [K]} b_{n,k} \tilde{x}_{n,k} - p^F_n) = (1-\epsilon') u^F_n \end{aligned}$$

We know u^F_n is the fractional VCG utility, which cannot be increased by a false bid. Because $u^F_n \geq 0$, we have $U^I_n \geq 0$. Hence the scaled-down integral payment is truthful in expectation. □

V. The Online C-RAN Auction Algorithm

We next design an online C-RAN auction algorithm towards near-optimal social welfare and truthfulness, using randomized rounding techniques.

A. Randomized One-Round Resource Allocation Algorithm

Algorithm 2 computes a tentative allocation solution for bidder n who arrives in round l. Line 1 computes the optimal solution to the scaled LP (2), with scaling factor $f = \frac{l}{N}$ where S is the set of hitherto arrived bidders till round l. Because the optimal solution $\tilde{x}^{(l)}$ is fractional, it is not directly applicable in practice. Thus, based on randomized rounding, we consider $\tilde{x}^{(l)}_{n,k}$ as the probability bidder n's option k wins. To satisfy constraint (1c), only one option $k \in [K]$ of bidder n can win. Therefore, Line 2 choose one option k with probability $\tilde{x}^{(l)}_{n,k}$. Subsequently, line 3 sets the corresponding variable $x_{n,k}$ of the randomly chosen option k to 1, all other variables to 0. Then in Line 4 we return the tentative allocation output $x^{(l)}$ as well as $\tilde{x}^{(l)}$.

Recall that the capacity ratio for our C-RAN system is C and h is the largest total number of BSs and links a mobile operator bids from. To be in line with real-world system settings, we assume our C-RAN system has a high capacity ratio $C = \Omega(\frac{\log h}{\epsilon^2})$. Next we bound the expected value contribution in one round to the overall social welfare of our C-RAN auction system.

Lemma 3: Let $S \subseteq [N]$ be a random subset of bids with $|S| = l$ and $l \geq 2\sqrt{\frac{1+\ln h}{C}} N$. Then we have

$$\mathbf{E}[\max_{x \in \mathcal{P}(\frac{l}{N}, S)} b^T x] \geq (1 - 9\sqrt{\frac{1+\ln h}{l/N \cdot C}}) \frac{l}{N} \max_{x \in \mathcal{P}(1,[N])} b^T x$$

Proof: Let x^* be the optimal solution to the LP relaxation of ILP (1), *i.e.*, $\max_{x \in \mathcal{P}(1,[N])} b^T x$. Consider round l of Algorithm 2, we construct a solution x' where $x'_{n,k} = x^*_{n,k}$ if $n \in S$, $x'_{n,k} = 0$ otherwise. Because the current bidder n arrives in random order, the probability that $n \in S$ is $\frac{l}{N}$. Therefore, we have $\mathbf{E}[x'_{n,k}] = \frac{l}{N} x^*_{n,k}$. However, x' may not be feasible to the scaled LP (2), which means that x' is not included in $\mathcal{P}(\frac{l}{N}, S)$. We can scale each $x'_{n,k}$ with a factor $F_{n,k}$ to $x''_{n,k}$, *i.e.*, $x''_{n,k} = F_{n,k} x'_{n,k}$. Then we can obtain a feasible solution x'' to the scaled LP (2). We set $F_{n,k} = \min\{1, \min_{m \in G^{(1)}_{n,k}} \frac{\frac{l}{N} R_m}{(rx')_m}, \min_{i \in G^{(2)}_{n,k}} \frac{\frac{l}{N} D_i}{(dx')_i}\}$ where $G^{(1)}_{n,k}$ and $G^{(2)}_{n,k}$ represent those BSs and fiber links operator n bids from in its option k, *i.e.*, $G^{(1)}_{n,k} = \{m \in [M] | r_{n,k,m} > 0\}, G^{(2)}_{n,k} = \{i \in [I] | d_{n,k,i} > 0\}, \forall (n,k) \in [N] \times [K]$. Let r denote $(r_{n,k,m}) \in \mathbb{R}^{N \times K \times M}$ and d denote $(d_{n,k,i}) \in \mathbb{R}^{N \times K \times I}$. Now x'' satisfies $rx'' \leq \frac{l}{N} R$ and $dx'' \leq \frac{l}{N} D$, where R is the vector of R_m, $m \in [M]$, and D is the vector of D_i, $i \in [I]$.

Fix a bidder $\tilde{n} \in [N]$. We first consider the conditional probability space when $\tilde{n} \in S$, *i.e.*, bidder n arrives before or at the current round l. Furthermore, pick a BS $m \in [M]$ and a fiber link $i \in [I]$ respectively. We will apply Chernoff bound on both random variable $(rx')_m$ and $(dx')_i$. First, for $(rx')_m$, define $X_n = \sum_{k \in [K]} r_{n,k,m} x'_{n,k}$ for each bid $n \in [N]$ which leads to $(rx')_m = \sum_{n \in [N]} X_n$. However, X_n is not independent. We need to consider their correlation [36]. The corresponding independent twin variable $\hat{X}_n$ is set to $\sum_{k \in [K]} r_{n,k,m} x^*_{n,k}$ with probability $\frac{l}{N}$ and 0 otherwise. It is clear that $\mathbf{E}[\sum_{n \in [N]} X_n | \tilde{n} \in S] \leq \mathbf{E}[\sum_{n \in [N]} \hat{X}_n | \tilde{n} \in S]$. Considering an arbitrary subset $\Pi \subseteq [N] \backslash \{\tilde{n}\}$ and some positive integers ϕ_n, we have,

$$\begin{aligned} &\mathbf{E}[\prod_{n \in \Pi} X^{\phi_n}_n | \tilde{n} \in S] \\ &\quad = \mathbf{E}[\prod_{n \in \Pi} (\sum_{k \in [K]} r_{n,k,m} x'_{n,k})^{\phi_n} | \tilde{n} \in S] \\ &\quad \leq \prod_{n \in \Pi} (\sum_{k \in [K]} r_{n,k,m} x^*_{n,k})^{\phi_n} (\frac{l}{N})^{|\Pi|} = \prod_{n \in \Pi} \mathbf{E}[\hat{X}^{\phi_n}_n | \tilde{n} \in S] \end{aligned}$$

Therefore, those random variables X_n are 1-correlated [36]. Moreover, because x^* is a feasible

LP solution and $r_{n,k,m} \le r_{\max}$, we get

$$\begin{aligned}
&\mathbf{E}[(rx')_m|\tilde{n} \in S] \\
&\quad = \mathbf{E}[\sum_{n\in[N]}\sum_{k\in[K]} r_{n,k,m}x'_{n,k}|\tilde{n} \in S] \\
&\quad = \sum_{n\in[N]}\sum_{k\in[K]} r_{n,k,m}\mathbf{E}[x'_{n,k}|\tilde{n} \in S] \\
&\quad = \frac{l}{N}\sum_{n\in[N]\setminus\{\tilde{n}\}}\sum_{k\in[K]} r_{n,k,m}x^*_{n,k} + \sum_{k\in[K]} r_{\tilde{n},k,m}x^*_{\tilde{n},k} \\
&\quad \le \frac{l}{N}R_m + r_{\max}
\end{aligned}$$

Therefore, we can apply a Chernoff bound [36] on $(rx')_m$ to get for all $0 \le \delta \le 1$,

$$\begin{aligned}
&\Pr\left[(rx')_m \ge (1+\delta)(\frac{l}{N}R_m + r_{\max})|\tilde{n} \in S\right] \\
&\quad \le \exp(-\frac{\delta^2}{3}(\frac{l}{N}R_m + r_{\max})) \le \exp(-\frac{\delta^2}{3}\frac{l}{N}R_{\min}) \quad (4)
\end{aligned}$$

Similarly, for the random variable $(dx')_i$, $\mathbf{E}[(dx')_i|\tilde{n} \in S] \le \frac{l}{N}D_i + d_{\max}$, for all $0 \le \delta \le 1$, applying the Chernoff bound on it, we will obtain that,

$$\begin{aligned}
&\Pr\left[(dx')_i \ge (1+\delta)(\frac{l}{N}D_i + d_{\max})|\tilde{n} \in S\right] \\
&\quad \le \exp(-\frac{\delta^2}{3}(\frac{l}{N}D_i + d_{\max})) \le \exp(-\frac{\delta^2}{3}\frac{l}{N}D_{\min}) \quad (5)
\end{aligned}$$

By the definition of capacity ratio $C = \min\{\frac{R_{\min}}{r_{\max}}, \frac{D_{\min}}{d_{\max}}\}$, we know $C \le \frac{R_m}{r_{\max}}$. Under the condition $l \ge \frac{2N}{\sqrt{C}}$, we can get $\frac{l}{N}\frac{R_m}{\sqrt{C}} \ge \frac{2R_m}{C} \ge 2r_{\max} \ge (1+\delta)r_{\max}$. Therefore,

$$\begin{aligned}
&\Pr\left[\frac{\frac{l}{N}R_m}{(rx')_m} \le \frac{1}{1+\delta+1/\sqrt{C}}|\tilde{n} \in S\right] \\
&\quad = \Pr\left[(rx')_m \ge (1+\delta+1/\sqrt{C})\frac{l}{N}R_m|\tilde{n} \in S\right] \\
&\quad \le \Pr\left[(rx')_m \ge (1+\delta)(\frac{l}{N}R_m + r_{\max})|\tilde{n} \in S\right] \quad (6)
\end{aligned}$$

Similarly, with $C \le \frac{D_i}{d_{\max}}$, $l \ge \frac{2N}{\sqrt{C}}$, we have,

$$\begin{aligned}
&\Pr\left[\frac{\frac{l}{N}D_i}{(dx')_i} \le \frac{1}{1+\delta+1/\sqrt{C}}|\tilde{n} \in S\right] \\
&\quad \le \Pr\left[(dx')_i \ge (1+\delta)(\frac{l}{N}D_i + d_{\max})|\tilde{n} \in S\right] \quad (7)
\end{aligned}$$

Applying the bounds (4) to (7), the definition $F_{n,k} = \min\{1, \min_{m\in G^{(1)}_{n,k}}\frac{\frac{l}{N}R_m}{(rx')_m}, \min_{i\in G^{(2)}_{n,k}}\frac{\frac{l}{N}D_i}{(dx')_i}\}$ and a union bound, we get

$$\begin{aligned}
&\Pr\left[F_{\tilde{n},k} \le \frac{1}{1+\delta+1/\sqrt{C}}|\tilde{n} \in S\right] \\
&\quad \le \sum_{m\in G^{(1)}_{\tilde{n},k}} \Pr\left[\frac{\frac{l}{N}R_m}{(rx')_m} \le \frac{1}{1+\delta+1/\sqrt{C}}|\tilde{n} \in S\right] \\
&\qquad + \sum_{i\in G^{(2)}_{\tilde{n},k}} \Pr\left[\frac{\frac{l}{N}D_i}{(dx')_i} \le \frac{1}{1+\delta+1/\sqrt{C}}|\tilde{n} \in S\right] \\
&\quad \le \sum_{m\in G^{(1)}_{\tilde{n},k}} \Pr\left[(rx')_m \ge (1+\delta)(\frac{l}{N}R_m + r_{\max})|\tilde{n} \in S\right] \\
&\qquad + \sum_{i\in G^{(2)}_{\tilde{n},k}} \Pr\left[(dx')_i \ge (1+\delta)(\frac{l}{N}D_i + d_{\max})|\tilde{n} \in S\right] \\
&\quad \le h_1\exp(-\frac{\delta^2}{3}\frac{l}{N}R_{\min}) + h_2\exp(-\frac{\delta^2}{3}\frac{l}{N}D_{\min}) \\
&\quad \le h\exp(-\frac{\delta^2}{3}\frac{l}{N}\Gamma) \quad (8)
\end{aligned}$$

Next, we need a lower bound on the expectation of $F_{\tilde{n},k}$. As $0 < F_{\tilde{n},k} \le 1$, we can partition the interval $(0, 1]$ into infinitely many subintervals with a step-width $\xi = \sqrt{3 \cdot \frac{1+\ln h}{l/N\cdot C}}$ and bound the probability that $F_{\tilde{n},k}$ is in each jth interval $\Delta_j = (\frac{1}{1+(j+1)\xi}, \frac{1}{1+j\xi}]$. We can lower-bound the expectation of $F_{\tilde{n},k}$ using the lower endpoint of an interval:

$$\begin{aligned}
\mathbf{E}[F_{\tilde{n},k}|\tilde{n} \in S] &\ge \sum_{j=0}^{\infty}\frac{1}{1+(j+1)\xi}\cdot\Pr[F_{\tilde{n},k} \in \Delta_j|\tilde{n} \in S] \\
&= \sum_{j=0}^{\infty}(1-\frac{(j+1)\xi}{1+(j+1)\xi})\cdot\Pr[F_{\tilde{n},k} \in \Delta_j|\tilde{n} \in S] \\
&= 1 - \sum_{j=0}^{\infty}\frac{(j+1)\xi}{1+(j+1)\xi}\cdot\Pr[F_{\tilde{n},k} \in \Delta_j|\tilde{n} \in S]
\end{aligned} \quad (9)$$

We consider the sum in three parts, $j \in \{0, 1, 2\}$, $j \in \{3, ..., \lfloor 1/\xi + 1\rfloor\}$ and $j \in \{\lfloor 1/\xi + 1\rfloor + 1, ...\}$. Because the sum of probabilities is at most 1 and the largest term is when $j = 2$, the sum for $j \in \{0, 1, 2\}$ is bounded by

$$\sum_{j\in\{0,1,2\}}\frac{(j+1)\xi}{1+(j+1)\xi}\cdot\Pr[F_{\tilde{n},k} \in \Delta_j|\tilde{n} \in S] \le 3\xi \quad (10)$$

For $j \in \{3, ..., \lfloor 1/\xi + 1\rfloor\}$, since $\delta \in (0, 1]$, we can set $\delta = (j-1)\xi$ and we have $\xi = \sqrt{3 \cdot \frac{1+\ln h}{l/N\cdot C}} \ge \frac{1}{\sqrt{C}}$. By assumption, we know both d_{max} and r_{max} are constants no less than 1, thus $\frac{\Gamma}{C} \ge 1$. Applying inequality (8), we have:

$$\begin{aligned}
&\Pr\left[F_{\tilde{n},k} \le \frac{1}{1+j\xi}|\tilde{n} \in S\right] \\
&\quad \le \Pr\left[F_{\tilde{n},k} \le \frac{1}{1+\delta+1/\sqrt{C}}|\tilde{n} \in S\right] \\
&\quad \le h\exp(-\frac{\delta^2}{3}\frac{l}{N}\Gamma) = h\exp(-\frac{(j-1)^2\xi^2}{3}\frac{l}{N}\Gamma) \\
&\quad \le h\exp(-(j-1)^2(1+\ln h)) \le \exp(-j+1) \quad (11)
\end{aligned}$$

Using inequality (11), we can bound the sum when $j \in \{3, ..., \lfloor 1/\xi + 1\rfloor\}$,

$$\begin{aligned}
&\sum_{j=3}^{\lfloor 1/\xi+1\rfloor}\frac{(j+1)\xi}{1+(j+1)\xi}\cdot\Pr\left[F_{\tilde{n},k} \in \Delta_j|\tilde{n} \in S\right] \\
&\quad \le \sum_{j=3}^{\lfloor 1/\xi+1\rfloor}(j+1)\xi\cdot\Pr\left[F_{\tilde{n},k} \le \frac{1}{1+j\xi}|\tilde{n} \in S\right] \\
&\quad \le \sum_{j=3}^{\lfloor 1/\xi+1\rfloor}(j+1)\xi\cdot\exp(-j+1) \le \frac{4e-3}{e(e-1)^2}\xi \le \xi \quad (12)
\end{aligned}$$

Finally, for $j \in \{\lfloor 1/\xi + 1 \rfloor + 1, ...\}$, because $\lfloor 1/\xi + 1 \rfloor \geq 3$, $\Gamma \geq C$, we know $\frac{l}{N}\Gamma \geq \frac{l}{N}C \geq 6\ln h$. Then using inequality (8) and $e^{-\frac{1}{2}x} \leq \sqrt{\frac{1}{x}}$, we can obtain the bound for the sum as follows:

$$\begin{aligned}\sum_{j=\lfloor 1/\xi+1\rfloor+1}^{\infty} & \frac{(j+1)\xi}{1+(j+1)\xi} \cdot \Pr[F_{\tilde{n},k} \in \Delta_j | \tilde{n} \in S] \\ & \leq \Pr[F_{\tilde{n},k} \leq \frac{1}{1+(\lfloor 1/\xi+1\rfloor+1)\xi} | \tilde{n} \in S] \\ & \leq \Pr[F_{\tilde{n},k} \leq \frac{1}{2+1/\sqrt{C}} | \tilde{n} \in S] \leq h\exp(-\frac{1}{3}\frac{l}{N}\Gamma) \\ & \leq \exp(-\frac{1}{6}\frac{l}{N}\Gamma) \leq \sqrt{\frac{3}{\frac{l}{N}\Gamma}} \leq \sqrt{3 \cdot \frac{1+\ln h}{l/N \cdot C}} = \xi \end{aligned} \tag{13}$$

Adding the three bounds (10) (12) (13), we obtain the lower bound of $\mathbf{E}[F_{\tilde{n},k}|\tilde{n} \in S]$ in inequality (9) with $\mathbf{E}[F_{\tilde{n},k}|\tilde{n} \in S] \geq 1 - 5\xi \geq 1 - 9\sqrt{\frac{1+\ln h}{l/N \cdot C}}$. Because $x''_{n,k} = F_{n,k}x'_{n,k}$ and $x'_{n,k} = x^*_{n,k}$ if $n \in S$, otherwise $x'_{n,k} = 0$, we have:

$$\begin{aligned}\mathbf{E}[x''_{n,k}] &= \Pr[n \in S]\mathbf{E}[F_{n,k}x'_{n,k}|n \in S] \\ &\quad + \Pr[n \notin S]\mathbf{E}[F_{n,k}x'_{n,k}|n \notin S] \\ &= \Pr[n \in S]\mathbf{E}[F_{n,k}|n \in S]x^*_{n,k} \\ &\geq (1 - 9\sqrt{\frac{1+\ln h}{l/N \cdot C}})\frac{l}{N}x^*_{n,k}, \forall n \in [N], k \in [K]\end{aligned}$$

Because x'' is a feasible solution of $\max_{x\in\mathcal{P}(\frac{l}{N},S)} b^T x$, we can derive the following bound:

$$\begin{aligned}\mathbf{E}[\max_{x\in\mathcal{P}(\frac{l}{N},S)} b^T x] &\geq \sum_{n\in[N]}\sum_{k\in[K]} b_{n,k}\mathbf{E}[x''_{n,k}] \\ &\geq (1 - 9\sqrt{\frac{1+\ln h}{l/N \cdot C}})\frac{l}{N}\sum_{n\in[N]}\sum_{k\in[K]} b_{n,k}x^*_{n,k} \\ &= (1 - 9\sqrt{\frac{1+\ln h}{l/N \cdot C}})\frac{l}{N}\max_{x\in\mathcal{P}(1,[N])} b^T x\end{aligned}$$

□

B. The Online Algorithm

After analyzing each round's potential contribution, we need to determine whether the temporary allocation for each round is feasible for the entire system. The allocation should not exceed the C-RAN system's resource capacity. The following online allocation algorithm, Algorithm 3, is designed to obtain the final allocation solution for all the N bidders and guarantee feasibility.

In Algorithm 3, y is the online allocation for the entire system. In each round, a bidder n arrives. Line 4 updates the currently known bidder set S and the round number l. Line 5 runs A_{round} and computes the tentative one-round solution $x^{(l)}$. Line 7 guarantees the feasibility of constraints (1a) and (1b). Then, if the temporary one-round result $x^{(l)}$ is feasible, y is updated in line 8. It is obvious that Algorithm 3 computes a feasible solution y. In Lemma 3, we already know the bound for each round's contribution. Next, we need to bound the probability of feasibility, *i.e.*, the probability a bidder n can be served with enough spectrum and bandwidth.

Algorithm 3 Online Allocation Algorithm for ILP (1)

1: Let $S := \emptyset$ be the index set of current known bidders;
2: Set $y := 0$;
3: **for** each arriving bidder n **do** // *step* $l = 1$ *to* N
4: Set $S := S \cup \{n\}$ and $l := |S|$;
5: Run $A_{round}(S, l, N, \{R_m\}, \{D_i\}, \{b_{n,k}, r_{n,k,m}, d_{n,k,i}\})$;
6: Get round l's output $\{\tilde{x}^{(l)}, x^{(l)}\}$;
7: **if** $r(y + x^{(l)}) \leq R$ and $d(y + x^{(l)}) \leq D$ **then**
8: $y = y + x^{(l)}$;
9: **end if**
10: **end for**

Lemma 4: Let $S \subseteq [N]$ be a random subset of bids with $|S| = l - 1$ and consider round l where $9\sqrt{\frac{1+\ln h}{C}}N \leq l \leq (1 - 9\sqrt{\frac{1+\ln h}{C}})N$. Let ε_S denote the event that bids in S come within the first $l - 1$ steps. Considering constraint (1a) and constraint (1b), the probability that a constraint $m \in [M]$ is violated is at most $\Pr[(\sum_{l'<l} rx^{(l')})_m > R_m - r_{max}|\varepsilon_S]$; similarly, the probability that a constraint $i \in [I]$ is violated is at most $\Pr[(\sum_{l'<l} dx^{(l')})_i > D_i - d_{max}|\varepsilon_S]$. We can bound these two probabilities respectively as follows,

$$\Pr[(\sum_{l'<l} rx^{(l')})_m > R_m - r_{max}|\varepsilon_S] \leq \frac{1}{h}\exp(-(1-\frac{l}{N})\sqrt{C})$$

$$\Pr[(\sum_{l'<l} dx^{(l')})_i > D_i - d_{max}|\varepsilon_S] \leq \frac{1}{h}\exp(-(1-\frac{l}{N})\sqrt{C})$$

Proof: First, for a constraint $m \in [M]$, we set $X_{l'} = (rx^{(l')})_m$ and the corresponding twin variable $\hat{X}_{l'}$ to 1 with probability $\frac{R_m}{N}$ and 0 otherwise. We can prove those random variables $X_{l'}$ are 1-correlated following the approach in [36]. Thus, we have $\mathbf{E}[X_{l'}] \leq \mathbf{E}[\hat{X}_{l'}]$ and $\mathbf{E}[\sum_{l'<l}\hat{X}_{l'}|\varepsilon_S] \leq \sum_{l'=1}^{l-1}\frac{R_m}{N} \leq \frac{l}{N}R_m$. We can now apply the Chernoff bound on $\sum_{l'<l} X_{l'}$.

$$\begin{aligned}\Pr[(\sum_{l'<l} & rx^{(l')})_m > R_m - r_{max}|\varepsilon_S] \\ &= \Pr[\sum_{l'<l} X_{l'} > R_m - r_{max}|\varepsilon_S] \\ &\leq \Pr[\sum_{l'<l} X_{l'} > (1+\delta)(1-\delta)(R_m - r_{max})|\varepsilon_S] \\ &\leq \exp(-\frac{\delta^2}{3}(1-\delta)(R_m - r_{max}))\end{aligned} \tag{14}$$

Because $9\sqrt{\frac{1+\ln h}{C}}N \leq l \leq (1 - 9\sqrt{\frac{1+\ln h}{C}})N$ and given the high capacity ratio C, we have $\frac{1}{9}(1 - \frac{l}{N}) \geq \frac{1}{\sqrt{C}} \geq \frac{1}{C-1}$. Setting $(1-\delta)(R_m - r_{max}) = \frac{l}{N}R_m \geq \mathbf{E}[\sum_{l'<l}\hat{X}_{l'}|\varepsilon_S]$, we can get $\delta = 1 - \frac{l}{N}\frac{R_m}{R_m - r_{max}}$. Thus,

$$\begin{aligned}\delta &\geq 1 - \frac{l}{N} - \frac{l}{N}\frac{1}{C_1 - 1} \geq 1 - \frac{l}{N} - \frac{l}{N}\frac{1}{C-1} \\ &\geq 1 - \frac{l}{N} - \frac{1}{C-1} \geq \frac{8}{9}(1 - \frac{l}{N}) \geq 8\sqrt{\frac{1+\ln h}{C}}.\end{aligned} \tag{15}$$

On the other hand, $1-\delta = \frac{l}{N}\frac{R_m}{R_m - r_{max}} \geq \frac{l}{N} \geq 9\sqrt{\frac{1+\ln h}{C}}$. Furthermore, using the fact that $\delta(1-\delta) \geq \frac{1}{2}\min\{\delta, 1-\delta\} \geq 4\cdot\sqrt{\frac{1+\ln h}{C}}$, we can obtain

$$\begin{aligned}\frac{\delta^2}{3}(1-\delta)(R_m - r_{max}) &\geq \frac{8}{9}(1-\frac{l}{N})\frac{4(R_m - r_{max})}{3}\sqrt{\frac{1+\ln h}{C}}\\ &\geq (1+\frac{1}{9})(1-\frac{l}{N})\sqrt{C(1+\ln h)}\\ &\geq \ln h + (1-\frac{l}{N})\sqrt{C} \qquad (16)\end{aligned}$$

Using the above inequality (16) we can bound the probability in Eqn. (14)

$$\begin{aligned}&\Pr[(\sum_{l'<l} rx^{(l')})_m > R_m - r_{max}|\varepsilon_S]\\ &\leq \exp(-\ln h - (1-\frac{l}{N})\sqrt{C})\\ &= \frac{1}{h}\exp(-(1-\frac{l}{N})\sqrt{C})\end{aligned}$$

For a constraint $i \in [I]$ in (1b), we can similarly obtain the bound for probability,

$$\Pr[(\sum_{l'<l} dx^{(l')})_i > D_i - d_{max}|\varepsilon_S] \leq \frac{1}{h}\exp(-(1-\frac{l}{N})\sqrt{C})$$

□

Therefore, we know that for an arriving bidder n, the probability that the C-RAN system may not have enough bandwidth or spectrum resource to serve it is at most $\frac{1}{h}\exp(-(1-\frac{l}{N})\sqrt{C})$. Based on Lemmas 3 and 4, we know the expectation and probability for one round l. We next bound the expected value of $b^T y$ we can get from Algorithm 3 by summing over all iterations.

Theorem 4: Assuming the number of BSs and links where a bidder bids resource from is at most h and the capacity ratio is C, the online Algorithm 3 achieves a $(1 - O(\sqrt{\frac{1+\ln h}{C}}))$ competitive ratio against offline optimal solution.

Proof: For convenience, we use q to denote the constant $9\sqrt{\frac{1+\ln h}{C}}$, *i.e.* $q = 9\sqrt{\frac{1+\ln h}{C}}$. We first consider the round l where $qN \leq l \leq (1-q)N$. Let $y^{(l)}$ be the change in round l, *i.e.* $y^{(l)} = x^{(l)}$ if the solution is feasible to ILP (1), otherwise $y^{(l)} = 0$.

From Lemma 3, we know the expectation of the optimal solution in round l is bounded as $\mathbf{E}[b^T\tilde{x}^{(l)}] \geq (1 - 9\sqrt{\frac{1+\ln h}{l/N\cdot C}})\frac{l}{N}\mathrm{OPT}^f$. Because we don't know the order of the first l bidders, we consider selecting one of the first l bidders uniformly at random as the l-th bidder. We can obtain:

$$\mathbf{E}[b^T x^{(l)}] \geq \frac{1}{l}\mathbf{E}[b^T\tilde{x}^{(l)}] \geq (1-9\sqrt{\frac{1+\ln h}{l/N\cdot C}})\frac{\mathrm{OPT}^f}{N}$$

From lemma 4, in round l we know the probability that one of the constraints in (1a) or (1b) is violated is at most $\frac{1}{h}\exp(-(1-\frac{l}{N})\sqrt{C})$. Because there are totally at most h constraints in (1a) and (1b) that are influenced by the l-th bidder in round l, applying the union bound, we can obtain the probability that the solution in round l is feasible *i.e.*, $y^{(l)} = x^{(l)}$, is at least $1-\exp(-(1-\frac{l}{N})\sqrt{C})$. Then we can bound the expected contribution to the objective function $b^T y^{(l)}$:

$$\begin{aligned}&\mathbf{E}[b^T y^{(l)}]\\ &\geq (1-\exp(-(1-\frac{l}{N})\sqrt{C}))\mathbf{E}[b^T x^{(l)}]\\ &\geq (1-\exp(-(1-\frac{l}{N})\sqrt{C}))(1-9\sqrt{\frac{1+\ln h}{l/N\cdot C}})\frac{\mathrm{OPT}^f}{N}\end{aligned}$$

The expectation of the online algorithm solution can be obtained by summing up all the rounds and we can bound it as follows,

$$\begin{aligned}&\mathbf{E}[\mathrm{ALG}]\\ &\geq \sum_{l=qN}^{(1-q)N}(1-\exp(-\frac{N-l}{N}\sqrt{C}))(1-9\sqrt{\frac{1+\ln h}{l/N\cdot C}})\frac{\mathrm{OPT}^f}{N}\\ &\geq \Bigg(1-2q-\sum_{l=qN}^{(1-q)N}\frac{1}{N}\exp(-\frac{N-l}{N}\sqrt{C})\\ &\qquad -\sum_{l=qN}^{(1-q)N}\frac{9}{N}\sqrt{\frac{1+\ln h}{l/N\cdot C}}\Bigg)\mathrm{OPT}^f \qquad (17)\end{aligned}$$

We separately bound the first and second sum components in Eqn. (17), using the inequality $1-\exp(-x) \geq (1-\frac{1}{e})x$ for all $0 \leq x \leq 1$ and $\sum_{i=1}^{n}\frac{1}{\sqrt{i}} \leq 2\sqrt{n}$. Thus, we obtain:

$$\begin{aligned}&\sum_{l=qN}^{(1-q)N}\frac{1}{N}\exp(-\frac{N-l}{N}\sqrt{C})\\ &\leq \frac{1}{N}\sum_{j=0}^{\infty}\exp(-\frac{j}{N}\sqrt{C})\\ &\leq \frac{1}{N(1-\exp(-\frac{\sqrt{C}}{N}))} \leq \frac{1}{(1-\frac{1}{e})\sqrt{C}} \leq q \qquad (18)\end{aligned}$$

For the second sum component, we have:

$$\begin{aligned}\sum_{l=qN}^{(1-q)N}\frac{9}{N}\sqrt{\frac{1+\ln h}{l/N\cdot C}} &\leq 9\sqrt{\frac{1+\ln h}{N\cdot C}}\sum_{l=1}^{N}\frac{1}{\sqrt{l}}\\ &\leq 18\sqrt{\frac{1+\ln h}{C}} \leq 2q \qquad (19)\end{aligned}$$

Finally, using Eqn. (17), (18) and (19), we derive:

$$\mathbf{E}[\mathrm{ALG}] \geq (1-5q)\mathrm{OPT}^f = (1-O(\sqrt{\frac{1+\ln h}{C}}))\mathrm{OPT}^f$$

Because the competitive ratio is defined as $\frac{\mathbf{E}[\mathrm{ALG}]}{\mathrm{OPT}}$ and we know $\mathrm{OPT} \leq \mathrm{OPT}^f$, we have $\frac{\mathbf{E}[\mathrm{ALG}]}{\mathrm{OPT}} \geq \frac{\mathbf{E}[\mathrm{ALG}]}{\mathrm{OPT}^f} \geq 1-O(\sqrt{\frac{1+\ln h}{C}})$. □

Therefore, our online algorithm can compute a close-to-optimal solution with a competitive ratio $1-O(\sqrt{\frac{1+\ln h}{C}})$. Given a high capacity ratio C in a practical C-RAN system, the competitive ratio is found to be around 0.8 in our performance evaluation studies. To evaluate the efficiency of our online algorithm, we next prove Algorithm 3 runs in polynomial time.

Algorithm 4 Truthful Online Algorithm for ILP (1)

1: Let $S := \emptyset$ be the index set of known requests;
2: Set $y := 0$;
3: **for** each arriving request n **do** // *step* $l = 1$ *to* N
4: Set $S := S \cup \{n\}$ and $l := |S|$;
5: **for** each option k such that $\exists m \in [M]$ with $(ry)_m + r_{n,k,m} > R_m$ or $\exists i \in [I]$ with $(dy)_i + d_{n,k,i} > D_i$ **do**
6: Set $b_{n,k} = 0$; // *remove infeasible options*
7: **end for**
8: Run $A_{round}(S, l, N, \{R_m\}, \{D_i\}, \{b_{n,k}, r_{n,k,m}, d_{n,k,i}\})$;
9: Get round l's output $\{\tilde{x}^{(l)}, x^{(l)}\}$;
10: Compute fractional VCG payment
11: $p_n^F = \max_{x \in \mathcal{P}(\frac{l}{N}, S\setminus\{n\})} b^T x - \sum_{n' \in S\setminus\{n\}} \sum_{k \in [K]} b_{n',k}\tilde{x}_{n',k}^{(l)}$;
12: Set $y := y + x^{(l)}$;
13: Charge the payment: $p_n^I = \frac{\sum_{k\in[K]} b_{n,k} x_{n,k}^{(l)}}{\sum_{k\in[K]} b_{n,k}\tilde{x}_{n,k}^{(l)}} p_n^F$
14: **end for**

Theorem 5: Algorithm 3 executes in polynomial time.

Proof: Because Algorithm 3 calls the one-round algorithm A_{round}, we first analyze the time complexity of A_{round}. In A_{round}, Line 1 computes the optimal solution to a scaled LP in polynomial time. Line 2 and Line 3 execute in $O(K)$ and $O(KN)$ respectively. Thus, A_{round} can be finished in polynomial time. The `for` loop in Algorithm 3 executes with each bidder' arrival. It will execute N iterations in total. We only need to prove Line 4 to 9 in the `for` loop run in polynomial time. Line 4 can be finished in $O(1)$. The `if` statement in Line 7 to 9 executes in $O(KN)$, linear to the number of bidders N and bidder's options K. Line 5 calls A_{round}. Therefore, Algorithm 3 runs in polynomial time. □

C. Truthful Payment Strategy

The previous analyses are all based on the assumption that all mobile operators bid with their true valuation. To guarantee truthfulness, we resort to a fractional version of the VCG auction for achieving both computational efficiency as well as truthfulness.

In Algorithm 4, along with each bidder's arrival, the for-loop from Line 5 to 7 is to check the feasibility and remove infeasible options for each bidder in advance through setting the infeasible bidding price to 0. Therefore, those infeasible solution will not be chosen when running A_{round} in Line 8. We first compute a fractional VCG payment p_n^F in Line 11 using the optimal fractional solution $\tilde{x}^{(l)}$ in each round. Then in Line 13, the auctioneer charges the payment p_n^I. We will then prove that the expectation of p_n^I equals p_n^F and p_n^I is no larger than bidder n's winning option k's valuation.

Theorem 6: Algorithm 4 computes the payment p_n^I for each bidder n, which can guarantee individual rationality and truthfulness in expectation.

Proof: In our online C-RAN auction, one bidder is served in each round. Suppose in round l, bidder n comes with K options and bids its true valuation $b_{n,k}$ for each option $k \in [K]$. The utility function u_n^F in the fractional VCG auction is defined as the difference between its valuation and the payment, *i.e.*, $u_n^F = \sum_{k\in[K]} b_{n,k}\tilde{x}_{n,k}^{(l)} - p_n^F$. In *Theorem 3*, we prove the fractional VCG payment is individually rational, *i.e.*, $u_n^F \geq 0$ and under the fractional VCG auction, the utility can not be increased by a falsified bid. Thus, the fractional VCG auction is truthful.

Then the fractional solution is scaled to integral solution by randomized rounding. $x_{n,k}^{(l)}$ is the binary variable which equals 1 with probability $\tilde{x}_{n,k}^{(l)}$. First we will show the utility under integral solution is also individually rational, *i.e.*, $u_n^I \geq 0$.

$$\begin{aligned} u_n^I &= \sum_{k\in[K]} b_{n,k} x_{n,k}^{(l)} - p_n^I \\ &= \sum_{k\in[K]} b_{n,k} x_{n,k}^{(l)} - \frac{\sum_{k\in[K]} b_{n,k} x_{n,k}^{(l)}}{\sum_{k\in[K]} b_{n,k}\tilde{x}_{n,k}^{(l)}} p_n^F \\ &= \frac{\sum_{k\in[K]} b_{n,k} x_{n,k}^{(l)}}{\sum_{k\in[K]} b_{n,k}\tilde{x}_{n,k}^{(l)}} \Big(\sum_{k\in[K]} b_{n,k}\tilde{x}_{n,k}^{(l)} - p_n^F\Big) \end{aligned}$$

Because $\sum_{k\in[K]} b_{n,k}\tilde{x}_{n,k}^{(l)} - p_n^F = u_n^F \geq 0$, we can get $u_n^I \geq 0$. For each bidder n, only one of the options k may be chosen with probability $\tilde{x}_{n,k}^{(l)}$. Then we calculate the expectation of the utility:

$$\begin{aligned} U_n^I &= \sum_{k\in[K]} \tilde{x}_{n,k}^{(l)} u_n^I \\ &= \sum_{k\in[K]} \tilde{x}_{n,k}^{(l)} \frac{\sum_{k\in[K]} b_{n,k} x_{n,k}^{(l)}}{\sum_{k\in[K]} b_{n,k}\tilde{x}_{n,k}^{(l)}} \Big(\sum_{k\in[K]} b_{n,k}\tilde{x}_{n,k}^{(l)} - p_n^F\Big) \\ &= \sum_{k\in[K]} x_{n,k}^{(l)} \frac{\sum_{k\in[K]} b_{n,k}\tilde{x}_{n,k}^{(l)}}{\sum_{k\in[K]} b_{n,k}\tilde{x}_{n,k}^{(l)}} \Big(\sum_{k\in[K]} b_{n,k}\tilde{x}_{n,k}^{(l)} - p_n^F\Big) \\ &= \sum_{k\in[K]} x_{n,k}^{(l)} \Big(\sum_{k\in[K]} b_{n,k}\tilde{x}_{n,k}^{(l)} - p_n^F\Big) = \sum_{k\in[K]} b_{n,k}\tilde{x}_{n,k}^{(l)} - p_n^F \end{aligned}$$

We know $\sum_{k\in[K]} b_{n,k}\tilde{x}_{n,k}^{(l)} - p_n^F$ is the utility under fractional VCG auction and it cannot be increased by false bid. Therefore, the scaled-down integral payment is truthful in expectation. □

VI. Performance Evaluation

We evaluate the proposed C-RAN mechanisms through simulation studies. We assume the number of BSs (M) is 10. Our C-RAN model in Fig. 1 supports both connection strategies, *i.e.*, a base station is connected to the VBS pool directly or through daisy chaining [37]. Therefore, the number of links is no more than the number of BSs. We choose $I = 6$ in our simulation. The bandwidth capacity of each link (D_i) is in the range of $[80, 200]$ units, and the spectrum capacity of each BS is set within $[120, 220]$ units. We vary the capacity values for different capacity ratio C. The number of bidders varies from 10 to 80, each submitting $K = 4$ options; the bidding price is based on true valuation of their resource requests. For each option, we assume the bidder picks some of the BSs with spectrum requests in the range of $[5, 10]$ units, and some of the fiber links with bandwidth consumption within $[10, 20]$ units.

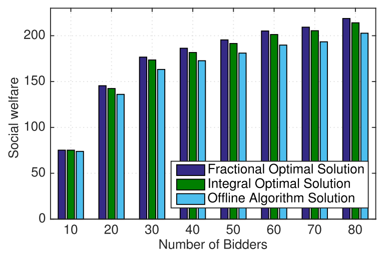

Fig. 2. Social Welfare comparison.

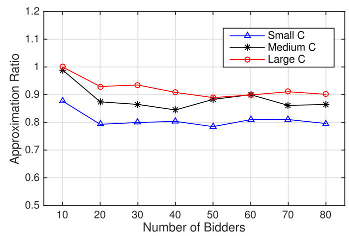

Fig. 3. Offline approximation ratio.

A. Performance of the Offline C-RAN Auction Algorithm

1) Social Welfare: To examine the performance of our offline approximation algorithm, we set the parameter $\epsilon' = 0.2$. First we execute Algorithm 1 under different number of bidders N. Then we compare the social welfare of our approximate algorithm with the integral and fractional optimum. As shown in Fig. 2, the gap between the optimum and the approximation solution is small regardless of the bidder population. The social welfare slightly increases with the number of number of bidders, because with a larger bidder pool, the auctioneer has a higher chance of identifying and selecting higher value bids.

2) Approximation Ratio: In Fig. 3, we further evaluate the approximation ratio of the offline algorithm, with 10 to 80 bidders, based on three levels of capacity ratio C. The ratio is mostly stable under varying bidder population, and remains at the level of 0.8 or higher. It can also be observed that, with a higher capacity ratio C, the algorithm can achieve a better approximate ratio. With the evidence shown in both Fig. 2 and Fig. 3, we can conclude that our offline approximate algorithm can compute an allocation solution that approaches optimal social welfare closely.

B. Performance of the Online C-RAN Algorithm

For benchmarking our online auction, we first compute the offline optimum of ILP (1) as well as its LP relaxation. Then, we execute our online algorithms, A_{round} and Algorithm 3, to obtain the approximate online solution to ILP (1) for comparison.

1) Competitive Ratio: The competitive ratio is defined as the ratio between the online solution and offline optimum. Fig. 4 shows that under different capacity ratio C and total requested number of BSs and links h, the competitive ratio increases when C grows and decreases when h rises. Such a trend agrees with the theoretical bound $1-O(\sqrt{(1+\ln h)/C})$. Furthermore, over the entire value space of C and h, the

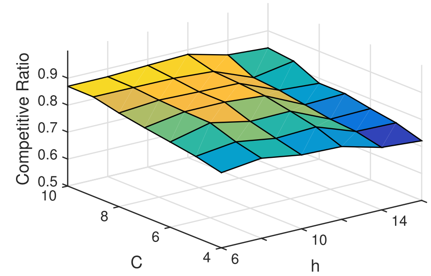

Fig. 4. Online competitive ratio.

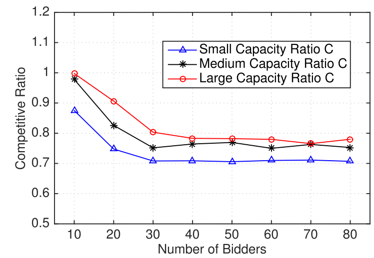

Fig. 5. Online competitive ratio.

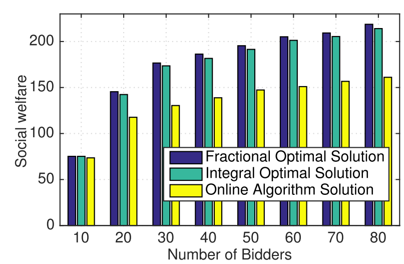

Fig. 6. Social Welfare comparison.

observed competitive ratio is always between 0.7 and 0.9. Therefore, it demonstrates that our online algorithm always achieve a relatively close to optimal performance under different C-RAN system settings. In Fig. 5, we can find that the competitive ratio is not influenced by the number of bidders when the number is large enough because the finite resources can only serve limited number of bidders; and a larger capacity ratio C leads to a higher competitive ratio. This is due to the fact that a large capacity ratio implies sufficient resource supply, hence the auctioneer can serve more bidders and make better choices.

2) Social Welfare: We further evaluate the social welfare and user satisfaction under different number of bidders. Fig. 6 compares the offline fractional optimal social welfare, offline integral optimal social welfare, and the online social welfare. It can be observed that our online algorithm closely approaches the two optimal values, with the fractional optimal social welfare being the highest. Generally, more bidders participating in the auction will lead to higher social welfare, since the C-RAN owner can have a larger pool of potential bids to select from.

3) Revenue and Utilities: We next evaluate our truthful payment strategy. Fig. 7 plots the seller revenue and bidders' utilities under different system settings. Recall that the sum of these two quantities is the social welfare of the C-RAN system.

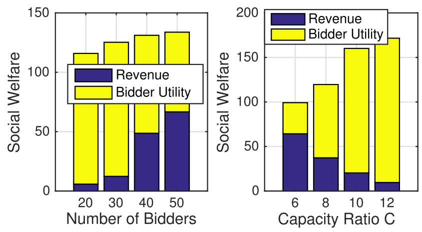

Fig. 7. Bidders' utility and revenue.

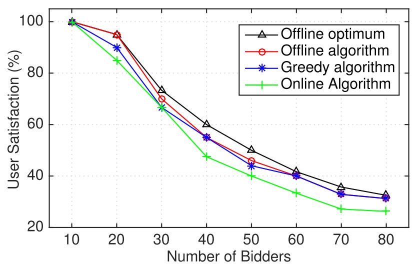

Fig. 8. User satisfaction level.

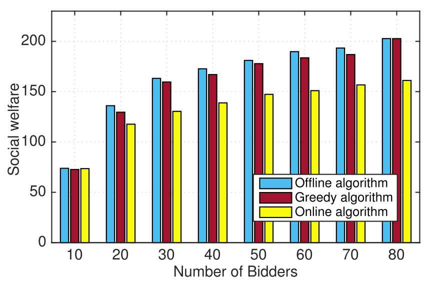

Fig. 9. Social Welfare comparison.

The left figure illustrates that more bidders leads to higher revenue as well as higher social welfare. As more bidders participate in the auction, the auctioneer has more choices to select from, leading to higher chances of high value bids winning the auction. As our payment strategy is based on the VCG mechanism, the winning bidder pays the externality it exerts on other bidders; more bidders participating to compete for limited resources results in to higher seller revenue. The right figure shows that along with the increase of the capacity ratio C, the social welfare increases while the revenue drops. This is because high capacity ratio means relatively abundant resources in the C-RAN system. Consequently, more bidders will be served, and higher social welfare is realized, meanwhile, the winning bidder's damage to other bidders will decrease, and the revenue will drop.

C. Performance Comparison: The Offline, Online C-RAN Auctions and the Greedy Solution

Zhu and Hossain [29] study combinatorial auction in 5G cellular networks and propose a greedy algorithm to solve the winner determination problem for the infrastructure provider and mobile virtual network operators. We implement their greedy algorithm to solve our problem and compare the performance with our offline and online auction mechanisms.

1) User Satisfaction: Fig. 8 plots *user satisfaction*, the percentage of winning bidders among all bidders. When the number of bidders rises, a decreasing trend is observed because the finite amount of resources can only serve a limited number of bidders. Our offline algorithm can achieve a better performance than the greedy algorithm. Moreover, the gap between the offline optimum and the online solution is small, which reveals that our online algorithm can satisfy a near-optimal number of bidders.

2) Social Welfare: Fig. 9 compares social welfare among the greedy, the offline and the online algorithms. Our offline algorithm achieves a higher social welfare than the greedy algorithm. The greedy algorithm simply decides the winners based on a bid's unit value. If a bid with high unit value consumes a large amount of resources, the greedy algorithm will still let it win, while it is harmful to the social welfare and user satisfaction. Because the online algorithm has no future information, it can perform worse than the offline algorithm. However, the difference is not substantial; our online algorithm can still achieve good performance.

VII. Conclusion

Driven by demands from the upcoming 5G wireless communication, C-RAN emerges as a new network infrastructure for managing large amounts of new spectrum and sites, reducing operating and capital expenses. This work is among the first that studies the C-RAN market where virtualized C-RAN resources are allocated to competing mobile operators through auction mechanisms. We designed near-optimal resource provisioning algorithms for both offline and online settings. We also make the C-RAN auction truthful using the fractional VCG strategy. As a future direction, one may study the mechanism design of the C-RAN market with delay-tolerant bids, which do not dictate immediate decision making.

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