An Online Procurement Auction for Power Demand Response in Storage-Assisted Smart Grids

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Abstract—The quintessential problem in a smart grid is the matching between power supply and demand — a perfect balance across the temporal domain, for the stable operation of the power network. Recent studies have revealed the critical role of electricity storage devices, as exemplified by rechargeable batteries and plug-in electric vehicles (PEVs), in helping achieve the balance through power arbitrage. Such potential from batteries and PEVs can not be fully realized without an appropriate economic mechanism that incentivizes energy discharging at times when supply is tight. This work aims at a systematic study of such demand response problem in storage-assisted smart grids through a well-designed online procurement auction mechanism. The long-term social welfare maximization problem is naturally formulated into a linear integer program. We first apply a primal-dual optimization algorithm to decompose the online auction design problem into a series of one-round auction design problems, achieving a small loss in competitive ratio. For the one round auction, we show that social welfare maximization is still NP-hard, and design a primal-dual approximation algorithm that works in concert with the decomposition algorithm. The end result is a truthful power procurement auction that is online, truthful, and 2-competitive in typical scenarios.

I. INTRODUCTION

The smart grid is a modern network for electric power generation, transportation and consumption that achieves high efficiency and robustness through sophisticated communication, control and optimization. The central problem in a smart grid, as in a traditional power network, is the realtime balancing between power supply and demand [1].

When the supply cannot match demand, a power grid may procure electricity from energy storage devices attached to the grid. For example, the power system in Ontario, Canada procured 35 megawatts of stored energy to provide ancillary services on May 5, 2014 [2]. When supply is higher than demand, electricity can be stored for future usage. Conventional electricity storage converts electricity immediately into another form of energy such as gravitational potential energy of water, compressed air energy, thermal energy and flywheel energy [3]. The paradigm is now changing with the emergence of new battery technologies. Rechargeable batteries are starting to play an important role in the electricity storage system. Lithium-ion batteries and liquid electrolyte "flow batteries" are two representative types of rechargeable battery used today. They can be found at grids, microgirds, electrical cars, smart

homes and individual customer sites. Batteries in grids and microgrids are usually used to stabilize the frequency of electric power. Electrical cars are also coming to the focal point as (i) new generation of car batteries can have 60kWh (about the average consumption of 30 U.S. households) or higher capacity [4], and (ii) the population of electrical cars rapidly grows. Smart homes are customarily equipped with battery to store electricity generated by the solar panel [5]. Individual customer sites such as datacenters and universities often own a large battery bank as well, for backup supply.
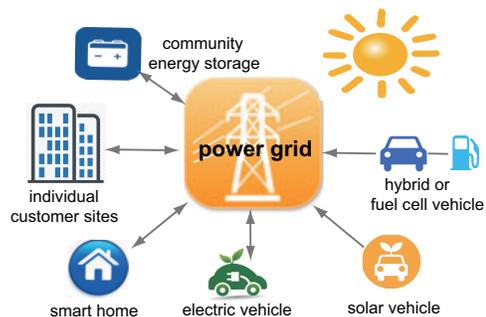
Fig. 1: A Storage-Assisted Smart Grid.

While a single battery may have limited capacity, thousands of them co-residing in the same grid can together store and supply an impressive amount of electricity. Power demand response through "storage crowdsourcing" is now envisioned as a critical tool towards balancing supply-demand in a power grid. It helps smooth demand and supply across the temporal domain via power arbitrage, by charging at night when the supply is high and the price is low (sometimes negative) and selling back to the gird when the demand is high. Rechargeable batteries are also more cost effective and environment friendly than quick-start generation (*e.g.*, diesel generators). For example, in *vehicle-to-grid* [6], a pilot project led by the University of Delaware, three types of cars are connected to the grid: hybrid and fuel cell vehicles, battery-powered vehicles, and solar vehicles. Each type is capable of producing 60 Hz AC electricity for either home consumption or selling back to the power grid through a connection line. Another ongoing community energy storage project at Toronto Hydro [7] installs a large battery box for each community, with 250kWh storage capacity. The storage helps keep voltage levels stable for commercial customers, removes the need for diesel generators and supports electricity releasing to the

grid during peak demand, through electricity procurement. *Ontario smart home roadmap* [5] indicates that solar panel and batteries will be embedded in most household by 2015. Fig. 1 illustrates the structure of a storage-assisted smart grid system, all the peripheral components can communicate with the power grid to participate in demand response.

To fully realize the potential of storage power demand response in practice, two types of challenges are to be addressed: *technical* and *economic*. While technical challenges of such demand response have been at the focal point of a series of recent studies [8], [9], this work represents one of the first studies on the economic side that provides the necessary financial catalyst for making power demand response a reality. The proposed solution for incentivizing storage participation and cost minimization in power demand response is an online procurement power auction. The *online* property of the auction captures the fact that storage based power arbitrage has diurnal cycles, and electricity stored at low-price hours is in finite supply. The *procurement* form of the auction captures the fact that power demand response auction is a reverse auction, with multiple sellers (storage devices) and a single buyer (the grid). Other properties pursued in the auction mechanism design include *computational efficiency*, *economic efficiency* (social cost minimization) and *truthfulness*.

We formulate the social welfare maximization problem, which becomes social cost minimization in our procurement auction, into an online optimization problem in the form of a natural integer linear program (ILP). We show that the most natural ILP formulation has an unbounded integrality gap, and augment it with a set of redundant 'flow-cover' constraints.

We design an online auction framework that decomposes the online auction into a series of one-round auctions, while guaranteeing a bounded additive loss in competitive ratio. Such decomposition is based on a primal-dual algorithm that works on the augmented social cost minimization ILP and its dual LP. Intuitively, it maintains dual variables that are used to scale power cost in the current round, and then conducts per-round cost minimization based on scaled costs. The framework assumes the existence of a one-round power demand response algorithm that is truthful, and provides both a primal and a dual solution with a bounded gap.

We proceed to design such a one-round auction algorithm that satisfies the above requirements. We design a primal-dual algorithm based on greedy dual ascending, which iteratively updates a pair of primal and dual solutions. While primal feasibility is automatically achieved, dual feasibility is not strictly enforced during the iterations and is achieved through posteriori dual fitting. We prove that both the approximation ratio and the primal-dual gap of the solutions are bounded by a small constant 2 — the ratios observed in simulation studies are even much better, mostly within $(1.0, 1.2)$ (Sec. VI). Furthermore, we prove that winner selection in this one-round algorithm satisfies an important *monotone* property, and as a result, can work in concert with a *critical value* based payment scheme to form a truthful auction.

In the rest of the paper, we review related literature in Sec. II, and introduce system model and background of the online demand response auction in Sec. III. Sec. IV focuses on the online auction framework design, assuming a truthful one-round auction that is designed and analyzed in Sec. V. Sec. VI presents simulation studies, and Sec. VII concludes the paper.

II. RELATED WORK

Storage-assisted smart grids have witnessed an increasing number of studies since a decade ago, starting from vehicle-to-grid systems. Kempton *et al.* [8] discuss technical requirements for realizing vehicle-to-grid systems, and calculate the resource size, availability, and economic potential of such systems. Johansen [9] studies prompt-charging of electric vehicles using AC. Through extensive experiments, he develops a low-cost AC-only fast-charging station. Gao *et al.* [10] propose a contract-based mechanism to incentivize electric vehicles to participate in power demand response. Different from the above literature, this work focuses on the demand response auction design in smart grids.

A series of auction mechanisms are recently designed for smart grids. Zhang *et al.* [11] study the electricity markets between power grids and micogrids by adopting a randomized auction framework. They consider both grid-to-microgrid and microgrid-to-grid markets, and focus on the Unit Commitment Problem (UCP). Ma *et al.* [12] study auction mechanism design when users buy energy from the provider in a smart grid. They show that appropriate pricing rules can guarantee truthful bidding. They use smart meters to record user's consumption information and patterns, and compute payments according to consumption credit. Ramachandran *et al.* use a hybrid optimization algorithm to design an auction for distributed energy resource management in smart grid operation [13]. All the above auctions are one-round instead of online, and do not have a target amount in electricity purchases.

Niv and Joseph [14] study online algorithm design via a primal-dual approach. They show that a number of practical problems can be solved using the basic approach developed for online packing-covering problems. Shi *et al.* [15] apply a similar primal-dual technique to an online cloud computing problem, for designing combinatorial auctions of virtual machine instances. TODA [16] represents a truthful online double auction for spectrum allocation in wireless networks. They assume that the arrival of user requests follows a potion distribution, and there is no budget or capacity limit.

III. SYSTEM MODEL AND PRELIMINARIES

A. System Model

Consider a storage-assisted smart grid system in which batteries store electricity. When the power grid predicts electricity shortage in an upcoming time period, it initiates demand response by soliciting electricity sales from the the agents (batteries) through a reverse auction, or a procurement auction.

Let $[X]$ denote the integer set $\{1, 2, \ldots, X\}$. The system runs in a time-slotted fashion across a time span of T time slots; the powder grid acts as the auctioneer, receiving bids

from agents during each slot. Following the diurnal pattern of power arbitrage, we further assume $T \leq 24$, for batteries are charged during low price periods in the night. Let M be the number of batteries connected with the power grid. At the beginning of each round, each agent $m \in [M]$ submits a set of K bids. Each bid is a pair $(e_{m,k}^{(t)}, c_{m,k}^{(t)})$, where $e_{m,k}^{(t)}$ is the amount of electricity (in kWh) agent m supplies in its kth bid, and $c_{m,k}^{(t)}$ is the cost of $e_{m,k}^{(t)}$, which models agent m's opportunity cost of not utilizing the same amount of energy by itself. $E^{(t)}$ and $L^{(t)}$ denote the power demand (in KW) at time t and the length (in hours) of each time slot. The grid needs $E^{(t)}L^{(t)}$ kWh to cover its shortage during t. At the end of each round, the auctioneer announces: (i) a binary number $x_{m,k}^{(t)}$ corresponding to each bid, with $x_{m,k}^{(t)} = 1$ indicating a successful bid and $x_{m,k}^{(t)} = 0$ an unsuccessful bid; (ii) a payment $P_m^{(t)}$ for each winning agent m.

Adopting the XOR-bidding language [15], we assume that each agent can win at most one bid in each round. Furthermore, the total energy a agent m supplies to the grid cannot exceed its battery's capacity C_m. The terms *time slot* and *round* are used interchangeably. Table I summarizes notation for ease of reference.

B. Truthful Procurement Auction Design

Let $b_{m,k}^{(t)}$ be the declared cost of $e_{m,k}^{(t)}$ in agent m's kth bid, $b_{-m}^{(t)}$ be the set of bids from agents except m. The utility of an agent m's kth bid is:

$$u_{m,k}(b_{m,k}^{(t)}, b_{-m}^{(t)}) = \begin{cases} P_m^{(t)} - c_{m,k}^{(t)} & \text{if } x_{m,k}^{(t)} = 1 \\ 0 & \text{otherwise} \end{cases}$$

Each agent is selfish and rational, with a natural goal of maximizing its own utility. Consequently, an agent may choose to manipulate its bid ($b_{m,k}^{(t)} \neq c_{m,k}^{(t)}$) if doing so leads to a higher utility. The auctioneer wishes to minimize its total cost when purchasing electricity from agents, for which it is important to know agents' truthful costs.

Definition 1. (Truthful auction) An auction is truthful if reporting the true cost $c_{m,k}^{(t)}$ for $e_{m,k}^{(t)}$ is the dominant strategy for any agent m at any time t: $u_{m,k}(c_{m,k}^{(t)}, b_{-m}^{(t)}) \geq u_{m,k}(b_{m,k}^{(t)}, b_{-m}^{(t)})$.

Definition 2. (Individual rationality & No positive transfers) Utility $u_{m,k} \geq 0$ and payment $P_m^{(t)} \geq 0$, $\forall m, k, t$. Agents always obtain non-negative utility, and never have to pay the auctioneer in the procurement auction [17].

The Vickrey-Clarke-Groves (VCG) mechanism [18] is a well-known type of auction that ensures truthful bidding. However, VCG mechanisms require an optimal solution to the winner determination problem. If the latter is NP-hard and an approximation algorithm is applied, VCG is not truthful [19]. The problem of demand response can be viewed as a generalization of the classic knapsack problem that is NP-hard [20]. We will rely on Myerson's characterization instead for truthful auction design, and employ an efficient approximation algorithm for winner determination.

Theorem 1. [21], [22] A reverse auction is truthful if and only if:

TABLE I: Summary of Notation

M	# of agents (batteries)	T	# of time slots
K	# of bids per agent	$[X]$	integer set $\{1, 2, \ldots, X\}$
$E^{(t)}$	power demand (in kilowatt) of the grid at time t		
$L^{(t)}$	length (in hours) of each time slot		
$e_{m,k}^{(t)}$	amount of energy (in kWh) agent m can supply at time t in kth bid		
$c_{m,k}^{(t)}$	cost of $e_{m,k}^{(t)}$		
$x_{m,k}^{(t)}$	agent m's kth bid successful (1) or not (0) at time t		
$P_m^{(t)}$	payment to agent m at time t		
C_m	the total capacity (in kWh) of agent m's battery		
$A^{(t)}$	a set $A^{(t)} = \{(m_1, k_1), (m_2, k_2), \ldots\}$, $\sum_{(m,k) \in A^{(t)}} e_{m,k}^{(t)} < E^{(t)}L^{(t)}$		
$E(A^{(t)})$	$E^{(t)}L^{(t)} - \sum_{(m,k) \in A^{(t)}} e_{m,k}^{(t)}$		
$e_{m,k}(A^{(t)})$	$\min\{e_{m,k}^{(t)}, E(A^{(t)})\}$		
$w_{m,k}^{(t)}$	the increased cost of $e_{m,k}^{(t)}$ in each round		
α	the approximation ratio of $\mathcal{A}_{one}$		

- The auction result $x_{m,k}^{(t)}$ is monotonically non-increasing in $c_{m,k}^{(t)}$;
- Winners are paid threshold payments: $P_m^{(t)} = c_{m,k}^{(t)}x_{m,k}^{(t)} + \int_{c_{m,k}^{(t)}}^{\infty} x_{m,k}^{(t)} dc_{m,k}^{(t)}$

IV. THE ONLINE AUCTION FRAMEWORK

We design an online auction framework that decomposes the online demand response winner determination problem (WDP) into one-round WDPs, We will formulate the online WDP and one round WDP in Sec. IV-A and Sec. IV-B respectively, then design the online algorithm in Sec. IV-C.

A. The Online Auction Problem

Under the assumption of truthful bidding, a natural integer linear program (ILP) formulation of the WDP for social welfare maximization is:

$$\text{Minimize} \sum_{t \in [T]} \sum_{m \in [M]} \sum_{k \in [K]} c_{m,k}^{(t)} x_{m,k}^{(t)} \tag{1}$$

Subject to:

$$\sum_{k \in [K]} x_{m,k}^{(t)} \leq 1, \ \forall m \in [M], \forall t \in [T] \tag{1a}$$

$$\sum_{m \in [M]} \sum_{k \in [K]} e_{m,k}^{(t)} x_{m,k}^{(t)} \geq E^{(t)}L^{(t)}, \ \forall t \in [T] \tag{1b}$$

$$\sum_{t \in [T]} \sum_{k \in [K]} e_{m,k}^{(t)} x_{m,k}^{(t)} \leq C_m, \ \forall m \in [M] \tag{1c}$$

$$x_{m,k}^{(t)} \in \{0, 1\}, \ \forall m \in [M], \forall t \in [T], \forall k \in [K] \tag{1d}$$

In a procurement auction, social welfare maximization is equivalent to social cost minimization [23]. Constraint (1a) implements the XOR bidding rule. (1b) guarantees that aggregate supply from successful bids sufficiently covers the grid's predicted power shortage. (1c) states that an agent's demand response participation is limited by its battery capacity.

The integrality gap between this ILP and its LP relaxation can be as bad as $E^{(t)}L^{(t)}$, as illustrated through the next example. Assume there is one time slot, and two agents each submits one bid, with $e_1 = EL - 1, c_1 = 0, e_2 = EL, c_2 = 1$,

and $C_1 = C_2 = 2EL$. The optimal LP solution sets $x_1 = 1, x_2 = 1/EL$ and has a cost of $1/EL$. The optimal ILP solution has to choose both agents and incurs a cost of 1. The integrality gap is EL.

The competitive ratio of our online auction framework partially depends on the integrality gap (Sec. IV-C). We augment the WDP formulation in IP (1) by introducing a number of redundant *flow-cover* type of inequalities [24]. Consider a set $A^{(t)} = \{(m_1, k_1), (m_2, k_2), \dots\}$ such that at time slot t, $\sum_{(m,k) \in A^{(t)}} e_{m,k}^{(t)} < E^{(t)} L^{(t)}$. Let's define $X^{(t)}$ as a large set that includes all possible $A^{(t)}$ at time t.

$$E(A^{(t)}) = E^{(t)} L^{(t)} - \sum_{(m,k) \in A^{(t)}} e_{m,k}^{(t)},$$

$$e_{m,k}(A^{(t)}) = \min\{e_{m,k}^{(t)}, E(A^{(t)})\}.$$

(1) can be reformulated into an equivalent optimization:

$$\text{Minimize} \sum_{t \in [T]} \sum_{m \in [M]} \sum_{k \in [K]} c_{m,k}^{(t)} x_{m,k}^{(t)} \tag{2}$$

Subject to: $\sum_{k \in [K]} x_{m,k}^{(t)} \leq 1, \ \forall m \in [M], \forall t \in [T] \tag{2a}$

$$\sum_{m \in [M]} \sum_{k \in [K]} e_{m,k}(A^{(t)}) x_{m,k}^{(t)} \geq E(A^{(t)}),$$

$$\forall (m,k) \notin A^{(t)}, \forall A^{(t)} \in X^{(t)}, \forall t \in [T] \tag{2b}$$

$$\sum_{t \in [T]} \sum_{k \in [K]} e_{m,k}^{(t)} x_{m,k}^{(t)} \leq C_m, \forall m \in [M] \tag{2c}$$

$$x_{m,k}^{(t)} \in \{0, 1\}, \ \forall m \in [M], \forall t \in [T], \forall k \in [K] \tag{2d}$$

By the property of set $A^{(t)}$, we can verify that every feasible solution to (1) is a feasible solution to (2), and vice versa. Introducing dual variables y, z, and s to constraints (2a), (2b) and (2c), respectively, we formulate the dual LP of (2):

$$\text{Max} \sum_{A^{(t)} \in X^{(t)}} \sum_{t \in [T]} z(A^{(t)}) E(A^{(t)}) - \sum_{m \in [M]} s_m C_m - \sum_{m \in [M]} \sum_{t \in [T]} y_m^{(t)} \tag{3}$$

Subject to:

$$\sum_{A^{(t)} \in X^{(t)} : (m,k) \notin A^{(t)}} z(A^{(t)}) e_{m,k}(A^{(t)}) \leq c_{m,k}^{(t)} + s_m e_{m,k}^{(t)} + y_m^{(t)},$$

$$\forall m \in [M], \forall t \in [T], \forall k \in [K] \tag{3a}$$

$$y_m^{(t)}, z(A^{(t)}), s_m \geq 0, \forall m \in [M], \forall t \in [T] \tag{3b}$$

In our online algorithm, we first assume that a truthful auction mechanism $\mathcal{A}_{one}$ is carried out at each time slot to solve one-slot demand response WDP, guaranteeing an approximation ratio α in social welfare. Then our proposed online algorithm can decompose the long-term auction into one round auctions according to the remaining capacity of each battery. It guarantees a bounded additive loss in competitive ratio during the translation.

B. One-Round Winner Determination Problem (WDP)

The one-round WDP for social welfare maximization is:

$$\text{Minimize} \sum_{m \in [M]} \sum_{k \in [K]} w_{m,k}^{(t)} x_{m,k}^{(t)} \tag{4}$$

Subject to:

$$\sum_{k \in [K]} x_{m,k}^{(t)} \leq 1, \forall m \in [M] \tag{4a}$$

$$\sum_{m \in [M]} \sum_{k \in [K]} e_{m,k}(A^{(t)}) x_{m,k}^{(t)} \geq E(A^{(t)}),$$

$$\forall (m,k) \notin A^{(t)}, \forall A^{(t)} \in X^{(t)} \tag{4b}$$

$$x_{m,k}^{(t)} \in \{0, 1\}, \forall m \in [M], \forall k \in [K] \tag{4c}$$

The above ILP includes the same constraints in IP (2) except the capacity constraint (2c). We modify the cost $c_{m,k}^{(t)}$ to a scaled cost $w_{m,k}^{(t)}$ according to the level of the remaining battery at agent m. The dual LP to (4) is:

$$\text{Maximize} \sum_{A^{(t)} \in X^{(t)}} z(A^{(t)}) E(A^{(t)}) - \sum_{m \in [M]} y_m^{(t)} \tag{5}$$

Subject to:

$$\sum_{A^{(t)} \in X^{(t)} : (m,k) \notin A^{(t)}} z(A^{(t)}) e_{m,k}(A^{(t)}) \leq w_{m,k}^{(t)} + y_m^{(t)},$$

$$\forall m \in [M], \forall k \in [K] \tag{5a}$$

$$y_m^{(t)}, z(A^{(t)}) \geq 0, \forall m \in [M] \tag{5b}$$

Where y and z are the same dual variables as in the dual of (2), and correspond to constraints (4a) and (4b), respectively. Table 2 is the correspondence relation between variables and constraints in the primal and dual LPs.

TABLE II: var-constraint correspondance in primal & dual LPs

Primal	(2a) & (4a)	(2b) & (4b)	(2c)	$x_{m,k}^{(t)}$
Dual	$y_m^{(t)}$	$z(A^{(t)})$	s_m	(3a) & (5a)

In Sec. V, we design a one-round α-approximation algorithm $\mathcal{A}_{one}$ that guarantees truthful bidding, individual rationality and no positive transfer. In Sec. IV-C, we assume the existence of such an auction, and present and focus on our online algorithm framework.

C. The Online Algorithm Framework

The main idea behind the online WDP algorithm design is to consider the current level of the remaining battery capacity. Different electricity supply at each round leads to different overall cost. When a bid with large supply $(e_{m,k}^{(t)})$ is accepted by the grid at an early stage, that agent loses the opportunity to participate in future demand response because its stored energy depletes, which in turn may force the grid to purchase electricity from an expensive alternative. The optimal strategy for the grid is striking a balance between cash-in on an agent's current bid and preserving that agent's demand response potential for the future. The intuition we follow when designing our online algorithm is to control the possibility of a supply $(e_{m,k}^{(t)})$ winning based on the residual storage level.

Along this direction, we adjust the cost in a bid from agent m according to its remaining capacity. A new variable $s_m^{(t)}$ is introduced for each $m \in [M]$. The initial value of $s_m^{(0)}$ is set to 0, which is then increased with the decease of the remaining battery capacity. The new cost $w_{m,k}^{(t)}$ is equal to

the original cost plus $e_{m,k}^{(t)}s_m^{(t-1)}$, and will be used in the one-round auctions. From the auctioneer's point of view, a bid with a smaller remaining capacity will have a larger cost, reducing its possibility of winning. The value of $s_m^{(t)}$ is increased for each winning bid (line 6) and remains intact otherwise (line 8), where $\gamma = \max_{m\in[M],k\in[K],t\in[T]}\{C_m/e_{m,k}^{(t)}\}$. In line 6, the value of $s_m^{(t)}$ is updated carefully, which is the key technique to achieve low additive loss in competitive ratio. In Algorithm 1, it is practical to assume that an agent submits its bids according to its remaining energy level in its battery. During each round of the auction, $e_{m,k}^{(t)}$ is at most *balance of* C_m. Furthermore, we set the value of dual variable s_m in dual problem (3) to the value of variable $s_m^{(t)}$ in Algorithm 1 after T time slots. In this way, we adjust the dual variables at each round to approach the optimal solution of dual problem in (3).

Algorithm 1: The Online Auction Framework $\mathcal{A}_{online}$

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1 $s_m^{(0)} := 0, \forall m \in [M]$;
2 **foreach** $1 \le t \le T$ **do**
3 $\quad w_{m,k}^{(t)} = c_{m,k}^{(t)} + e_{m,k}^{(t)}s_m^{(t-1)}, \forall m \in [M]$;
4 $\quad$ Run Algorithm $\mathcal{A}_{one}$ at time slot t. Let's define $\mathcal{I}$ as the set of winning agents, and for each $m \in \mathcal{I}$, k_m as the index of their corresponding winning bid;
5 $\quad$ **foreach** $m \in \mathcal{I}$ **do**
6 $\quad\quad s_m^{(t)} = s_m^{(t-1)}\Big(1 + \frac{e_{m,k_m}^{(t)}}{\alpha C_m}\Big) + \frac{c_{m,k_m}^{(t)}}{\alpha\gamma C_m}$;
7 $\quad$ **foreach** $m \notin \mathcal{I}$ **do**
8 $\quad\quad s_m^{(t)} = s_m^{(t-1)}$;
9 $s_m = s_m^{(T)}, \forall m \in [M]$;

Theorem 2. Assume there is an approximation algorithm $\mathcal{A}_{one}$ for one-round WDP, providing feasible solutions to (4) and (5), and guaranteeing $\alpha d \ge p$. Note that the approximation ratio of $\mathcal{A}_{one}$ is also α. (By LP duality, $\alpha d \ge p \ge p*, p/p* \le p/d \le \alpha$). Then $\mathcal{A}_{online}$ is a $\alpha\frac{\gamma}{\gamma-1}$-approximation algorithm for the optimization problem in (2) and (1). Here p is the objective value of problem (4) achieved by $\mathcal{A}_{one}$, d is the dual objective value in (5) calculated by $\mathcal{A}_{one}$, and $p*$ is the optimal objective value for problem (4).

Proof: We first prove the following three claims:

Claim 1: $\mathcal{A}_{online}$ can produce a feasible solution for IP (2).

Proof of Claim 1: Because $\mathcal{A}_{one}$ running at each round can provide a feasible solution for problem in (4), then $x_{m,k}^{(t)}$ satisfies constraints (2a), (2b) and (2d). For the constraint (2c), we know that one agent cannot submit a bid that exceeds its remaining battery capacity. That is $e_{m,k}^{(t)} \le$ *balance of* C_m, therefore, constraint (2c) can be guaranteed. □

Claim 2: At the end of the execution of $\mathcal{A}_{online}$, it can output a feasible solution for dual problem in (3).

Proof of Claim 2: From the line 3 of Algorithm 1, we know that $w_{m,k}^{(t)}$ is equal to $c_{m,k}^{(t)} + e_{m,k}^{(t)}s_m^{(t-1)}$, also from constraint (5a) and the no-decreasing property of $s_m^{(t)}$ with

t, the following can be obtained:

$$\sum_{A^{(t)}\in X^{(t)}:(m,k)\notin A^{(t)}} z(A^{(t)})e_{m,k}(A^{(t)}) \le w_{m,k}^{(t)} + y_m^{(t)}$$

$$\sum_{A^{(t)}\in X^{(t)}:(m,k)\notin A^{(t)}} z(A^{(t)})e_{m,k}(A^{(t)}) \le c_{m,k}^{(t)} + e_{m,k}^{(t)}s_m^{(t-1)} + y_m^{(t)}$$

$$\sum_{A^{(t)}\in X^{(t)}:(m,k)\notin A^{(t)}} z(A^{(t)})e_{m,k}(A^{(t)}) \le c_{m,k}^{(t)} + s_m^T e_{m,k}^{(t)} + y_m^{(t)}$$

$s_m = s_m^{(T)}$, so (3a) holds, and (3b) also holds. □

Claim 3: Let's define $\Delta P^{(t)} = P^{(t)} - P^{(t-1)}$, where $P^{(t)}$ is the objective value of ILP (2) returned by $\mathcal{A}_{online}$, the same for $\Delta D^{(t)}$ in dual (3). Then at any time slot t, $\Delta P^{(t)} \le \alpha\frac{\gamma}{\gamma-1}\Delta D$ holds in $\mathcal{A}_{online}$.

Proof of Claim 3: at time slot t, $\Delta P^{(t)} = \sum_{m\in\mathcal{I}}c_{m,k_m}^{(t)}$

$$\Delta D^{(t)} = \sum_{m\in\mathcal{I}} C_m(s_m^{(t-1)} - s_m^{(t)}) + d$$

$$= d - \frac{\sum_{m\in\mathcal{I}}e_{m,k_m}^{(t)}s_m^{(t-1)}}{\alpha} - \frac{\sum_{m\in\mathcal{I}}c_{m,k_m}^{(t)}}{\alpha\gamma}$$

$$\ge \frac{p}{\alpha} - \frac{\sum_{m\in\mathcal{I}}e_{m,k_m}^{(t)}s_m^{(t-1)}}{\alpha} - \frac{\sum_{m\in\mathcal{I}}c_{m,k_m}^{(t)}}{\alpha\gamma}$$

$$\ge \frac{\sum_{m\in\mathcal{I}}w_{m,k_m}^{(t)}}{\alpha} - \frac{\sum_{m\in\mathcal{I}}e_{m,k_m}^{(t)}s_m^{(t-1)}}{\alpha} - \frac{\sum_{m\in\mathcal{I}}c_{m,k_m}^{(t)}}{\alpha\gamma}$$

$$\ge (\frac{1}{\alpha} - \frac{1}{\alpha\gamma})\Delta P^{(t)}.$$

□

Since $P^{(0)} = 0, D^{(0)} = 0$, and $\Delta P^{(t)} \le \alpha\frac{\gamma}{\gamma-1}\Delta D^{(t)}$ by Claim 3, thus $P^{(T)} \le \alpha\frac{\gamma}{\gamma-1}D^{(T)}$. By LP duality, $\mathcal{A}_{online}$ is a $\alpha\frac{\gamma}{\gamma-1}$ approximation algorithm for the optimization problem in (2). Furthermore, by the validity of the flow-cover inequalities every feasible solution to (2) is a feasible solution to (1), so $\mathcal{A}_{online}$ is a $\alpha\frac{\gamma}{\gamma-1}$-approximation algorithm for the optimization problem in (1) as well. □

Note that when $\gamma \to \infty$, *i.e.*, when agents are only interested in supplying a small proportion of their battery capacity, the competitive ratio approaches α, which means there is zero loss compare to the one-round algorithm.

V. THE ONE ROUND DEMAND RESPONSE AUCTION

In this section, we design a polynomial-time approximation algorithm for one-round WDP (4) in Sec. V-A, and tailor a payment scheme to work in concert with the algorithm to form a truthful auction.

A. A Primal-Dual Approximation Algorithm

Algorithm 2 is an iterative primal-dual algorithm based on dual ascending, solving simultaneously the primal problem (4) and its dual (5). The classic dual fitting technique is employed for *ex post* dual feasibility. Algorithm 2 elevates dual variable $z(A^{(t)})$ until dual constraint (5a) becomes tight, then adds the corresponding item to a set $A^{(t)}$, and increases the new dual variables again. This iterative process is repeated until items in $A^{(t)}$ can cover the grid's predicted gap in power supply. Bids selected in the final set constitute the solution to (4).

More specifically, lines 1-5 initialize the primal and dual variables, and the scaled cost $w_{m,k(m)}^{(t)}\star$. A `while` loop in lines 7-15 updates the primal and dual variables. It has two stop conditions. The first one ($[M] \neq \emptyset$) prevents infinite loops. The second one ensures the algorithm ends with a feasible solution to (4). Scaled cost is equal to the slacks of the dual constraints and is decremented at line 7 during each iteration. The algorithm selects a bid with the minimum scaled cost per effective value $e_{m,k}(A^{(t)})$ at line 8, and the corresponding primal variable $x_{m^*,k_m}^{(t)}$ is updated. In line 12, we don't increase dual variable $z(A^{(t)})$ slowly; instead we set its value to a minimum such that the dual constraint becomes tight. Line 10 is used to find the threshold bid (j, j_m), for which (m^*, k_m) is guaranteed to win as long as $\frac{w_{m^*,k_m}^{(t)}\star}{e_{m^*,k_m}(A^{(t)})} \leq \frac{w_{j,j_m}^{(t)}\star}{e_{j,j_m}(A^{(t)})}$. Then we calculate the cost to match (j, j_m) in line 13, and pay that amount to agent m^* for ensuring truthfulness (see Theorem 4). m^* is removed at line 15 to make sure no other bids of agent m^* wins in the future. Line 16 records the final allocation in set $\mathcal{I}$. The last two steps (line 17-18) generate a feasible dual solution to (5).

Algorithm 2: A Primal-Dual Approximation Algorithm $\mathcal{A}_{one}$ for One-round WDP (4)

1 $z(A^{(t)}) := 0, \forall A^{(t)} \in X^{(t)}$;
2 $A^{(t)} := \emptyset$;
3 $y_m^{(t)} := 0, \forall m \in [M]$;
4 $x_{m,k}^{(t)} := 0, \forall m \in [M], \forall k \in [K]$;
5 $w_{m,k}^{(t)}\star := w_{m,k}^{(t)}, \forall m \in [M], \forall k \in [K]$;
6 **while** $[M] \neq \emptyset$ **AND** $E(A^{(t)}) > 0$ **do**
7 $\quad w_{m,k}^{(t)}\star = w_{m,k}^{(t)}\star - z(A^{(t)})e_{m,k}(A^{(t)})$;
8 $\quad m^*, k_m := \arg\min_{m \in [M], k \in [K]}\{\frac{w_{m,k}^{(t)}\star}{e_{m,k}(A^{(t)})}\}$;
9 $\quad x_{m^*,k_m}^{(t)} := 1$;
10 $\quad j, j_m := \arg\min_{(m,k) \neq (m^*,k_m)}\{\frac{w_{m,k}^{(t)}\star}{e_{m,k}(A^{(t)})}\}$;
11 $\quad z(A_j) := \min_{(m,k) \neq (m^*,k_m)}\{\frac{w_{m,k}^{(t)}\star}{e_{m,k}(A^{(t)})}\}$;
12 $\quad z(A^{(t)}) := \min_{m \in [M], k \in [K]}\{\frac{w_{m,k}^{(t)}\star}{e_{m,k}(A^{(t)})}\}$;
13 $\quad P_{m^*}^{(t)} := w_{m^*,k_m}^{(t)} + (z(A_j) - z(A^{(t)}))e_{m^*,k_m}(A^{(t)})$;
14 $\quad A^{(t)} := A^{(t)} \cup (m^*, k_m)$;
15 $\quad [M] := [M] \setminus m^*$;
16 $\mathcal{I} := A^{(t)}$;
17 $\epsilon^{(t)} := \max\left\{\frac{w_{m_2,k_2}^{(t)}}{w_{m_1,k_1}^{(t)}}, \frac{e_{m_1,k_1}^{(t)}w_{m_2,k_2}^{(t)}}{e_{m_2,k_2}^{(t)}w_{m_1,k_1}^{(t)}}\right\}$;
18 $z(A^{(t)}) := z(A^{(t)})/\epsilon^{(t)}$;

B. Feasibility and Gap of Primal & Dual Solutions

Theorem 3. $\mathcal{A}_{one}$ terminates with a feasible solution for primal problem (4) and dual problem (5), and guarantees $\alpha d \geq p$ (d and p defined in Theorem 1), $\alpha = 2\epsilon^{(t)}$ with $\epsilon^{(t)} = \max(w_{m_2,k_2}^{(t)}/w_{m_1,k_1}^{(t)}, e_{m_1,k_1}^{(t)}w_{m_2,k_2}^{(t)}/e_{m_2,k_2}^{(t)}w_{m_1,k_1}^{(t)})$, $\forall m_1, m_2 \in [M], \forall k_1, k_2 \in [K]$. Therefore, $\mathcal{A}_{one}$ is a α-approximation algorithm. If we further restrict that each agent submits one bid only, the approximation ratio is 2.

Proof of Theorem 3: We first analyze the complexity of $\mathcal{A}_{one}$. One of the termination conditions of the `while` loop is $[M] = \emptyset$, therefore the loop iterates at most M times. Lines 7, 8, 10, 11 and 12 can be done within MK times. We conclude that the total running time is $O(MK)$, linear to the input size, and $\mathcal{A}_{one}$ runs in polynomial time. Next, we prove the correctness of Algorithm 2.

Claim 1: Upon termination, $\mathcal{A}_{one}$ generates a feasible solution to the primal problem (4).
Proof of Claim 1: The body of Algorithm 2 keeps adding a new item to set $A^{(t)}$ as long as $E(A^{(t)}) > 0$. Once we finish, all the items in $\mathcal{I}$ form our integer solution. Because $E(A^{(t)}) \leq 0$, $\sum_{(m,k) \in \mathcal{I}} e_{m,k}^{(t)} \geq E^{(t)}L^{(t)}$, which satisfies constraint (4b). Moreover, line 12 will exclude agent m^* from $[M]$, so other bids of m^* cannot be selected any more. As a result, constraint (4a) can be guaranteed. Constraint (4c) will not be violated because the value of $x_{m,k}^{(t)}$ is in initialized to 0 and updated to 1 at line 9, so its value can only be 0 or 1.

The algorithm also stops when $[M] = \emptyset$. In this case, we cannot not find a subset of agents that can meet the grid's demand. This is a less likely in practical scenarios where the number of agents is large. $\quad\square$

Claim 2: After the last iteration of the `while` loop in Algorithm $\mathcal{A}_{one}$, line 18 ensures that $z(A)/\epsilon$ is a feasible solution to dual problem (5).
Proof of Claim 2: at the beginning of Algorithm 2, dual variables are set to zero and are feasible. At the τ-th iteration,

$$z(A^\tau)e_{m^*,k_m}(A^\tau) = w_{m^*,k_m}^\tau\star$$
$$= w_{m^*,k_m}^{\tau-1}\star - z(A^{\tau-1})e_{m^*,k_m}(A^{\tau-1})$$
$$= w_{m^*,k_m}^{\tau-2}\star - z(A^{\tau-2})e_{m^*,k_m}(A^{\tau-2}) - z(A^{\tau-1})e_{m^*,k_m}(A^{\tau-1})$$
$$w_{m^*,k_m}^1\star = z(A^1)e_{m^*,k_m}(A^1) + \cdots + z(A^\tau)e_{m^*,k_m}(A^\tau)$$

Line 5 sets $w_{m^*,k_m}^1\star = w_{m^*,k_m}$, and at the end of τ-th iteration, bid (m^*, k_m) is appended to $A^{(t)}$. We have:

$$\sum_{A \in X:(m^*,k_m) \notin A} z(A)e_{m^*,k_m}(A)$$
$$= z(A^1)e_{m^*,k_m}(A^1) + \cdots + z(A^\tau)e_{m^*,k_m}(A^\tau) = w_{m^*,k_m}$$

The above equation can be applied to all $(m, k) \in \mathcal{I}$, that is

$$\sum_{A \in X:(m,k) \notin A} z(A)e_{m,k}(A) = w_{m,k}, \forall (m, k) \in \mathcal{I}.$$

Case 1: $(m, k) \in \mathcal{I}$. Recall that ϵ is defined as the maximum ratio that must be at least 1, and we have:

$$\frac{1}{\epsilon} \sum_{A \in X:(m,k) \notin A} z(A)e_{m,k}(A) \leq w_{m,k}, \forall (m, k) \in \mathcal{I}.$$

Case 2: $(m, k) \notin \mathcal{I}$. If we define

$$\epsilon = \max_{m_1,m_2 \in [M], k_1,k_2 \in [K]} \frac{e_{m_1,k_1}(A)w_{m_2,k_2}}{e_{m_2,k_2}(A)w_{m_1,k_1}}$$
$$\Rightarrow \frac{1}{\epsilon} \frac{e_{m_1,k_1}(A)}{w_{m_1,k_1}} \leq \frac{e_{m_2,k_2}(A)}{w_{m_2,k_2}}$$

We can further apply the above inequality as: for any $(m_1, k_1) \notin \mathcal{I}, m_2, k_2 \in \mathcal{I}$,

$$\sum_{A \in X : (m_1,k_1) \notin A} z(A) \frac{e_{m_1,k_1}(A)}{w_{m_1,k_1} \epsilon} \leq \sum_{A \in X : (m_2,k_2) \notin A} z(A) \frac{e_{m_2,k_2}(A)}{w_{m_2,k_2}} = 1$$

$$\Rightarrow \frac{1}{\epsilon} \sum_{A \in X : (m_1,k_1) \notin A} z(A) e_{m_1,k_1}(A) \leq w_{m_1,k_1}$$

Now we can see that (5a) will not be violated once $z(A) = z(A)/\epsilon$. The final step is to compute the value of ϵ.

$$\epsilon = \max_{m_1,m_2 \in [M], k_1, k_2 \in [K]} \frac{e_{m_1,k_1}(A)}{e_{m_2,k_2}(A)} \frac{w_{m_2,k_2}}{w_{m_1,k_1}}$$

Since $e_{m,k}(A) = \min\{e_{m,k}, E(A)\}$, if

i) $e_{m_1,k_1}(A) = e_{m_2,k_2}(A) = E(A), e_{m_1,k_1}(A)/e_{m_2,k_2}(A) = 1$;

ii) $e_{m_1,k_1}(A) = e_{m_1,k_1}, e_{m_2,k_2}(A) = e_{m_2,k_2}$,
$e_{m_1,k_1}(A)/e_{m_2,k_2}(A) = e_{m_1,k_1}/e_{m_2,k_2}$;

iii) $e_{m_1,k_1}(A) = E(A), e_{m_2,k_2}(A) = e_{m_2,k_2}$,
$e_{m_1,k_1}(A)/e_{m_2,k_2}(A) \leq e_{m_1,k_1}/e_{m_2,k_2}$;

iv) $e_{m_1,k_1}(A) = e_{m_1,k_1}, e_{m_2,k_2}(A) = E(A)$,
$e_{m_1,k_1}(A)/e_{m_2,k_2}(A) \leq E(A)/E(A) = 1$.

In summary, for $\forall m_1, m_2 \in [M], \forall k_1, k_2 \in [K]$:

$$\frac{e_{m_1,k_1}(A)}{e_{m_2,k_2}(A)} = \max\left\{1, \frac{e_{m_1,k_1}}{e_{m_2,k_2}}\right\},$$

$$\epsilon = \max\left\{\frac{w_{m_2,k_2}}{w_{m_1,k_1}}, \frac{e_{m_1,k_1} w_{m_2,k_2}}{e_{m_2,k_2} w_{m_1,k_1}}\right\}.$$

$\square$

Claim 3: The solutions to (4) and (5) returned by $\mathcal{A}_{one}$ guarantee $2\epsilon^{(t)} d \geq p$.

Proof of Claim 3: Let l be the last bid selected by Algorithm $\mathcal{A}_{one}$. In $\mathcal{A}_{one}$, $y_m^{(t)}$ is zero. Since the algorithm only selects an agent when (5a) becomes tight, the total cost is

$$p = \sum_{m \in [M]} \sum_{k \in [K]} w_{m,k}^{(t)} x_{m,k}^{(t)} = \sum_{(m,k) \in \mathcal{I}} w_{m,k}^{(t)}$$

$$= \sum_{(m,k) \in \mathcal{I}} \sum_{A^{(t)} \in X^{(t)} : (m,k) \notin A^{(t)}} z(A^{(t)}) e_{m,k}(A^{(t)})$$

$$= \sum_{A^{(t)} \in X^{(t)}} z(A^{(t)}) \sum_{(m,k) \in \mathcal{I} : (m,k) \notin A^{(t)}} e_{m,k}(A^{(t)})$$

$$\leq \sum_{A^{(t)} \in X^{(t)}} z(A^{(t)}) \left(\sum_{(m,k) \in \mathcal{I} \setminus l} e_{m,k}^{(t)} - \sum_{(m,k) \in A^{(t)}} e_{m,k}^{(t)} + e_l(A^{(t)}) \right)$$

Since $E(\mathcal{I} \setminus l) = E^{(t)} L^{(t)} - \sum_{(m,k) \in \mathcal{I} \setminus l} e_{m,k}^{(t)} > 0$, we have $\sum_{(m,k) \in \mathcal{I} \setminus l} e_{m,k}^{(t)} < E^{(t)} L^{(t)}$. Then

$$p \leq \sum_{A^{(t)} \in X^{(t)}} z(A^{(t)}) \left(E^{(t)} L^{(t)} - \sum_{(m,k) \in A^{(t)}} e_{m,k}^{(t)} + e_l(A^{(t)}) \right)$$

$$\leq \sum_{A^{(t)} \in X^{(t)}} z(A^{(t)}) \left(E^{(t)} L^{(t)} - \sum_{(m,k) \in A^{(t)}} e_{m,k}^{(t)} + E(A^{(t)}) \right)$$

$$\leq \sum_{A^{(t)} \in X^{(t)}} z(A^{(t)}) \left(E(A^{(t)}) + E(A^{(t)}) \right)$$

$$\leq 2\epsilon^{(t)} \sum_{A^{(t)} \in X^{(t)}} z(A^{(t)}) E(A^{(t)}) / \epsilon^{(t)} \leq 2\epsilon^{(t)} d.$$

$\square$

When each agent submits one bid only ($k = 1$), our algorithm can provide a feasible solution for both primal problem (4) and dual problem (5). $p = \sum_{(m,k) \in \mathcal{I}} w_{m,k}^{(t)} \leq 2 \sum_{A^{(t)} \in X^{(t)}} z(A^{(t)}) E(A^{(t)}) \leq 2d$. Hence the approximation ratio under this scenario is 2.

Claims 1-3 together imply the theorem. $\square$

C. Truthfulness of the Online Procurement Auction

Lemma 1. The auction result $x_{m,k}^{(t)}$ computed by $\mathcal{A}_{one}$ is **monotone**: $\forall m_1, m_2 \in [M], \forall k_1, k_2 \in [K]$, if $w_{m_1,k_1}^{(t)} \leq w_{m_2,k_2}^{(t)}$ and $e_{m_1,k_1}^{(t)} = e_{m_2,k_2}^{(t)}$, then $x_{m_2,k_2}^{(t)} = 1$ implies $x_{m_1,k_1}^{(t)} = 1$.

Proof of Lemma 1:
In Algorithm 2, line 8 can be reformulated as : $m*, k_m = \arg\min_{m \in [M], k \in [K]}\left\{ \frac{w_{m,k}^{(t)} - \sum_{A^{(t)} \in X^{(t)} : (m,k) \notin A^{(t)}} z(A^{(t)}) e_{m,k}(A^{(t)})}{e_{m,k}(A^{(t)})} \right\}$.
Because $e_{m_1,k_1}^{(t)} = e_{m_2,k_2}^{(t)}$, $e_{m_1,k_1}(A^{(t)}) = e_{m_2,k_2}(A^{(t)})$. We also have $w_{m_1,k_1}^{(t)} \leq w_{m_2,k_2}^{(t)}$, thus

$$\frac{w_{m_1,k_1}^{(t)} - \sum_{A^{(t)} \in X^{(t)} : (m_1,k_1) \notin A^{(t)}} z(A^{(t)}) e_{m_1,k_1}(A^{(t)})}{e_{m_1,k_1}(A^{(t)})}$$

$$\leq \frac{w_{m_2,k_2}^{(t)} - \sum_{A^{(t)} \in X^{(t)} : (m_2,k_2) \notin A^{(t)}} z(A^{(t)}) e_{m_2,k_2}(A^{(t)})}{e_{m_2,k_2}(A^{(t)})}.$$

Consequently, if our algorithm selects (m_2, k_2) at the τ-th iteration, it must select (m_1, k_1) first, so $x_{m_2,k_2}^{(t)} = 1$ implies $x_{m_1,k_1}^{(t)} = 1$. $\square$

Lemma 2. The payment scheme of $\mathcal{A}_{one}$ is individual rational and has no positive transfer.

Proof of Lemma 2: Let examine the line 13 of Algorithm 2, the auctioneer pays $P_{m^*}^{(t)} = w_{m^*,k_m}^{(t)} + \left(z(A_j) - z(A^{(t)})\right) e_{m^*,k_m}^{(t)}$ to agent m^*. Because $z(A_j) \geq z(A^{(t)})$, then $P_{m^*}^{(t)} \geq w_{m^*,k_m}^{(t)}$, therefor the utility $u_{m^*,k_m} \geq 0$ and $P_{m^*}^{(t)} \geq 0$. $\square$

Theorem 4. The auction performed by $\mathcal{A}_{one}$ is truthful.

Proof of Theorem 4: By Myerson's characterization of truthful mechanisms, an auction is truthful iff (i) the auction result is monotone, and (ii) winners are paid threshold payments. We already demonstrated the first property of our mechanism in Lemma 1. Next, we will explain the second property.

We know that bid (j, j_m) is the threshold bid for m^*, k_m, because when we exclude (m^*, k_m) from the candidates set, (j, j_m) would be first bid that gets assigned. Clearly, if $\frac{w_{m^*,k_m}^{(t)} \star}{e_{m^*,k_m}(A^{(t)})} \geq \frac{w_{j,j_m}^{(t)} \star}{e_{j,j_m}(A^{(t)})}$, (m^*, k_m) would not be the winner bid. So we can compute the payment to agent m^* such that it makes $\frac{w_{m^*,k_m}^{(t)} \star}{e_{m^*,k_m}(A^{(t)})} = \frac{w_{j,j_m}^{(t)} \star}{e_{j,j_m}(A^{(t)})}$, that is at iteration τ:

$$w_{m^*,k_m}^{\tau} \star = e_{m^*,k_m}(A^{\tau}) z(A_j)$$

Go back to the proof of Claim 2 in Theorom 3, we have:

$$w_{m*,k_m} = z(A^1) e_{m^*,k_m}(A^1) + \cdots + z(A^{\tau}) e_{m^*,k_m}(A^{\tau})$$

$$P_m^* = z(A^1) e_{m^*,k_m}(A^1) + \cdots + z(A_j) e_{m^*,k_m}(A^{\tau})$$

$$P_m^* = w_{m^*,k_m} + \left(z(A_j) - z(A^{\tau})\right) e_{m^*,k_m}(A^{\tau})$$

That is the same as $P_{m^*}^{(t)} := w_{m^*,k_m}^{(t)} + \left(z(A_j) - z(A^{(t)})\right) e_{m^*,k_m}(A^{(t)})$. $\square$

Theorem 5. $\mathcal{A}_{online}$ coupled with $\mathcal{A}_{one}$ is a a truthful, individual rational auction that has no positive transfer. It achieves $2\epsilon^{(t)}\frac{\gamma}{\gamma-1}$ approximation in social cost.

Combining Theorem 1, Lemma 2 and Theorem 4, we can prove Theorem 5. We observe that when $\gamma \to \infty$ and $k = 1$, the competitive ratio approaches 2.

VI. SIMULATION STUDIES

This section contains simulation studies of the one-round auction algorithm and the online auction framework. The grid demand $E^{(t)}$ is set between 10GW and 50GW, with reference to information from `ieso` (Power to Ontario) [2]. The length of each time slot $L^{(t)}$ is an hour. Battery capacity is chosen between 60kWh and 200kWh, following the capacity of current vehicle batteries and storage batteries. The amount of supply by one agent $e_{m,k}^{(t)}$ is a random value from $[0,100]$kWh. Magnitudes of cost $c_{m,k}^{(t)}$ are set with reference to cost information from a real-world utility, ENMAX [25], and follow a uniform distribution over $[\$0, \$20]$.

A. Performance of One-round Demand Response Auction

TABLE III: Comparison of the total cost ($) when $k = 1$

# agents	1000	1400	1800	2200	2600	3000
$\mathcal{A}_{one}$	407.74	301.99	231.82	202.77	154.28	132.99
Optimal	405.36	301.83	230.32	202.52	152.72	131.67

Theorem 3 shows that when an agent is restricted to submit a single bid per round, the approximation ratio of one-round auction $\mathcal{A}_{one}$ is reduced to 2. We plot the social cost under this scenario in Fig. 2, and list the cost values in table 3. In this figure, the energy shortage of the grid is equal to 10GWh. We can see that the solution returned by Algorithm 2 has a sightly higher cost than that of the optimal solution. Our simulations suggest that the one-round primal-dual algorithm can approach optimal solution much better than indicated by proven theoretical worst-case ratio 2. Furthermore, when the number of agents varies from 1000 to 3000, we find that the total cost decreases with the increase of the number of agents. This is because the gird can select more cost effective agents.
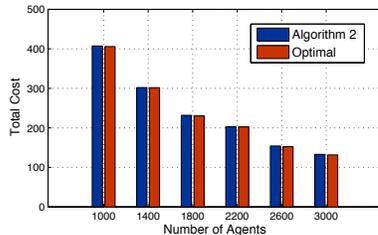
Fig. 2: Performance of $\mathcal{A}_{one}$ when $k = 1$.

We evaluate the performance of $\mathcal{A}_{one}$ in terms of approximation ratio, social cost and percentage of winners in Fig. 3. The 3d plot in Fig. 3a demonstrates the fluctuation of approximation ratio with the change of number of agents and number of bids per agent. We can see that the peek point of the approximation ratio appears when a small number of agents are available to participate in the demand response procedure,

and each user submits more than 10 bids. Furthermore, the ratio approaches 1 towards the bottom-right corner of the surface, which shows that our one-round algorithm performs close to optimal in social cost with a large number of agents and a small number of bids per agent. Even with small numbers of agents and large values of k, the algorithm can still achieve a rather impressive approximation ratio ($\sim$1.2).

Fig. 3b shows that total cost under different levels of demand with the increase of k. It reflects a downward trend as the number of bids per agent grows, while there is no large difference under varying demand. The underlying reason is the grid can select the best bid from one agent, and we set the number of agents to 2000, which can provide sufficient supply. Fig. 3c shows the ratio between the number of winnings agents and the number of total agents. The observation is that a large number of agents lends to a low percentage of winners for the same demand 20GW. Under the same agent population, the percentage of winners remains at the same level regardless of the value of k. This is true since for the same level of demand, the number of winners remains the same, but the probability of winning drops with the increasing of number of competitors.

B. Performance of Online Auction

We next study the performance of our online auction framework with $\mathcal{A}_{online}$ assisted by $\mathcal{A}_{one}$ running at each time slot.

Fig. 4a shows the total cost over different number of agents with $k = 2$ and $T = 4$. We label the cost for each round with different color and the height of the stack bar is the overall cost. Similar to the case of one-round auctions, the larger number of available agents, the better performance in terms of cost can be achieved. Then we examine the overall competitive ratio under different number of rounds in Fig. 4b. Small values in k and T lead to a lower ratio, which can be explained as following: the competitive ratio is related to the value of γ and ϵ, where γ is the maximum ratio between battery capacity and its supply in one bid, and ϵ is the maximum value of cost ratio and supply per cost ratio. When we have multiple rounds and bids per agent, the value of γ and ϵ is more likely to be a large value. The observation in Fig. 4c is that the our algorithm performs better with an increasing number of agents and decreasing value of k.

Finally, we examine the additive loss in competitive ratio of the online algorithm, as compared to the one-round algorithm, in Fig. 5. We can see that there are only a slight loss, which confirms our theoretical analysis in Theorem 2.

VII. CONCLUSIONS

Power arbitrage by timed charging and discharging electricity storage devices such as batteries and PEVs is proving an effective tool for reducing grid-wide power cost, guaranteeing balanced power demand and supply, and improving stability of modern smart grids. Besides engineering challenges that are being addressed in the literature, economic sides of such demand response in smart grids is also critical for realizing practical applications. This work represents one of the first studies on storage power demand response through an online

(a) Approximation Ratio

(b) Social cost

(c) Percentage of Winners
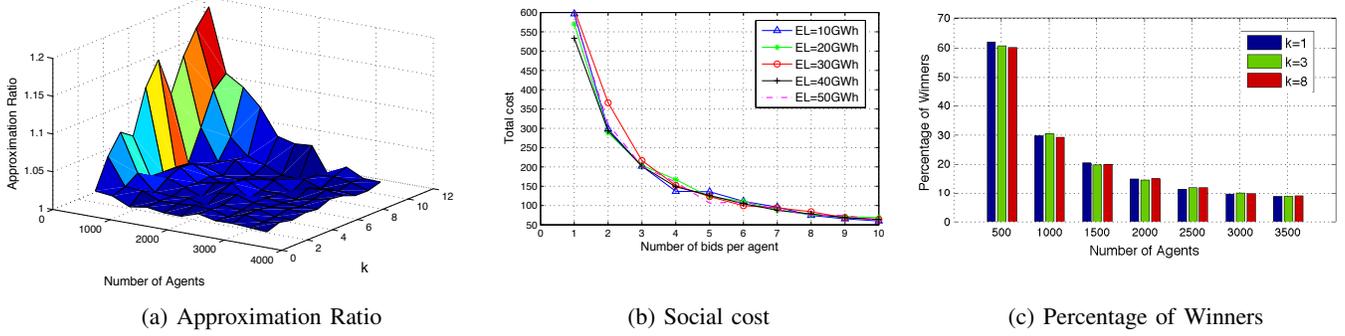
Fig. 3: Performance of one-round WDP algorithm under different settings.

(a) Total cost

(b) Competitive ratio, different T

(c) Competitive ratio, different M
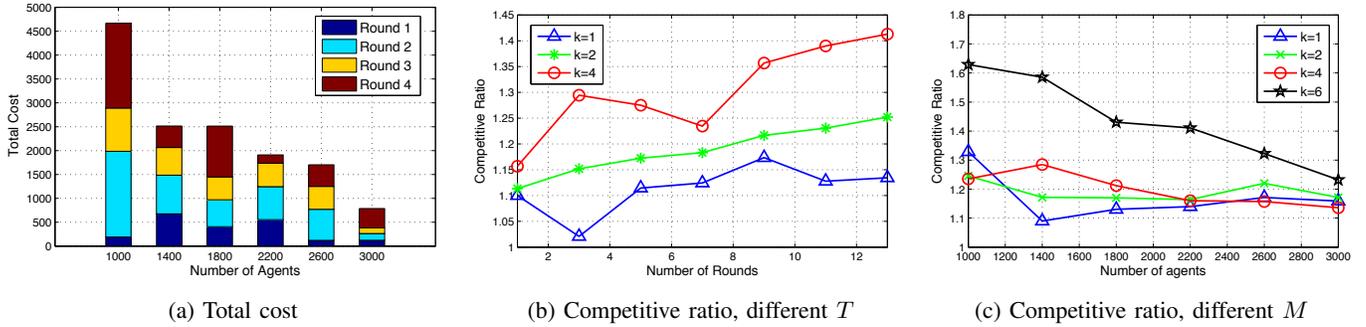
Fig. 4: Performance of online algorithm under different settings.
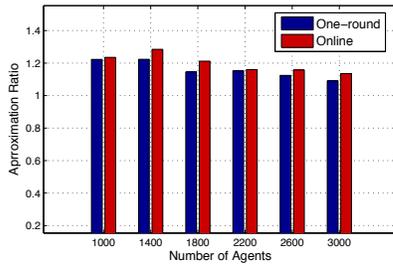
Fig. 5: Comparison of the approximation ratio between Algorithm 2 and Algorithm 1.

procurement power auction mechanism. The two-stage auction designed is truthful, computationally efficient, and achieves a competitive ratio of 2 in practical scenarios.

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