

Randomized Auction Design for Electricity Markets between Grids and Microgrids

Linquan Zhang
Dept. of Computer Science
University of Calgary
linqzhan@ucalgary.ca

Zongpeng Li
Dept. of Computer Science
University of Calgary
zongpeng@ucalgary.ca

Chuan Wu
Dept. of Computer Science
The University of Hong Kong
cwu@cs.hku.hk

ABSTRACT

This work studies electricity markets between power grids and microgrids, an emerging paradigm of electric power generation and supply. It is among the first that addresses the economic challenges arising from such grid integration, and represents the first power auction mechanism design that explicitly handles the Unit Commitment Problem (UCP), a key challenge in power grid optimization previously investigated only for centralized cooperative algorithms. The proposed solution leverages a recent result in theoretical computer science that can decompose an optimal fractional (infeasible) solution to NP-hard problems into a convex combination of integral (feasible) solutions. The end result includes randomized power auctions that are (approximately) truthful and computationally efficient, and achieve small approximation ratios for grid-wide social welfare under UCP constraints and temporal demand correlations. Both power markets with grid-to-microgrid and microgrid-to-grid energy sales are studied, with an auction designed for each, under the same randomized power auction framework. Trace driven simulations are conducted to verify the efficacy of the two proposed inter-grid power auctions.

Categories and Subject Descriptors

C.4 [Performance of Systems]: Design studies; Modeling techniques; I.1.2 [Algorithms]: Analysis of algorithms

General Terms

Algorithms, Design, Economics

Keywords

Power Grid; Microgrids; Unit Commitment Problem; Mechanism Design; Approximation Algorithms

1. INTRODUCTION

An electrical power grid is an interconnected network for delivering electricity from generators/suppliers to consumers. In a classic

power grid, electrical power is produced at (often remote) generating stations, travels through long-distance high-voltage transmission lines to demand centers, and then through distribution lines to end customers. A *microgrid*, in contrast, is a distributed electric power system, operating autonomously to organize local generation to meet the demand dynamically [19], e.g., a university served by its own power generators, or a hospital supplied mainly by its local generators. An important trend in power grid evolution is characterized by the rapidly increasing percentage of power supplied from microgrids. Such a paradigm shift has been jointly driven by a number of related factors: (a) The *economies of scale* associated with massive central generation starts to fail, as generation units and plants become cheaper to build, and transmission cost catches up with generation cost; (b) Concerns from grid customers on externalized costs of central plant generation and lack of billing control; (c) Environmental concerns, given that classic grids are often driven by non-renewable resources (coal, gas, nuclear), while latest microgrids commonly resort to wind and solar energy that are green and renewable [17]; (d) Reliability concerns, as exemplified by the 2003 Eastern US/Canada major blackout and the largest power outage in history (India, 2012), most power failures nowadays are traced back to the transmission grid instead of generation units [1]; (e) Even with traditional fuel, co-generation in residential microgrids provides both electricity and heat, bringing new opportunities for thermodynamically efficient use of fuel [23].

As a latest example, IKEA acquired a 46 MW wind farm in Alberta, Canada in November 2013, towards its eventual power self-sustainability based on green energy. Prior to that, the furniture giant already owns similar wind farms and solar generation plants in Europe and Eastern Canada, respectively. Microgrids have recently witnessed an impressive growing rate, and are estimated to further grow at a 17% compound annual rate in terms of installed capacity [4]. Global installed capacity is projected to reach 15 GW by 2022, roughly equivalent to the total installed capacity in Portugal in 2008.

For reliability reasons, a microgrid is usually connected to the main power grid in its region (Fig. 1), where the distribution stations reside. Temporary deficits in energy supply can be resolved by purchase from the regional grid; conversely, excess energy generated can be contributed to the regional grid. Such integration of a large number of microgrids to the traditional power grid represents a fundamental revolution in the energy sector, and it is generally agreed that a number of challenges from both technical and economic sides are to be addressed [1]. On the technical side, a plethora of research has been devoted to the study of power quality, voltage stability, harmonics, and grid-wide control and reliability [10, 19, 21, 30], leading to a relatively clear picture for issues and solutions regarding such grid integration. In contrast, the area of

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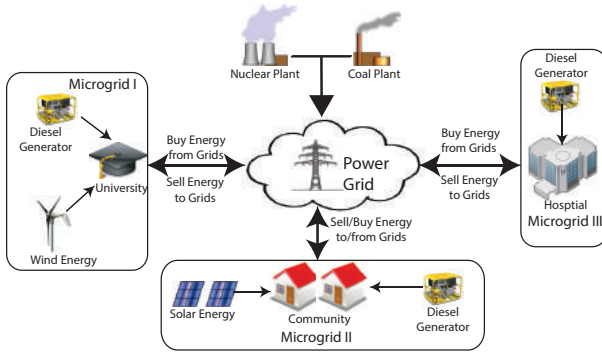


Figure 1: A regional power grid and microgrids.

market analysis and inter-grid energy trading mechanisms have essentially remained blank, witnessing few if any dedicated studies, despite its practical necessity towards smooth flowing of energy between microgrids and the main power grid.

The lack of research activity is partly attributed to the hardness of the problem: any sound economic mechanism design here must appropriately model and handle the Unit Commitment Problem (UCP), engineering constraints of generation units that impose strict limitations on their scheduling and operation. Most generators today, including traditional thermal (coal, gas, petroleum), nuclear, and renewable (wind, solar, hydro, wave-power, geothermal energy), have generator-specific parameters that define (a) minimum and maximum stable output levels, (b) maximum rates of ramping up or down, and (c) minimum time a unit stays ON or OFF. Along with the optimal power flow problem (OPF), UCP is known as one of the two key problems at the core of power grid optimization [28]. A series of studies in the past few decades investigate the modelling and solution of UCP, usually through a mixed linear integer programming model and centralized approximation algorithms [28]. A market mechanism such as an auction has to further consider economic behaviours of the two participating sides, such as individual rationality and truthful bidding. The challenge further escalates considering that a grid’s bid for electricity in practice is across multiple consecutive and correlated time slots (*e.g.*, for each of the next 24 hours) [29].

To the authors’ knowledge, this work is among the first that designs trading mechanisms for the grid-microgrid electricity market, towards the goal of designing power auctions that explicitly models and handles UCP and temporal demand correlations. Our solution combines (i) two carefully formulated mixed linear integer programming formulations of the social welfare optimization problems in the inter-grid market; (ii) a recent result in theoretical computer science, which decomposes a fractional (infeasible) optimal solution to packing and covering-type NP-hard problems into a convex combination of integral (feasible) solutions, enabling a randomized auction framework that translates centralized co-operative approximation algorithms into (approximately) truthful and computationally efficient auction mechanisms; and (iii) custom-designed approximation algorithms for social welfare optimization in the inter-grid market, which are computationally efficient, have small approximation ratios, and work in concert with the auction framework in (ii).

As a market mechanism, auctions enable market efficiency and agility through resource pricing based directly on realtime supply-demand. Compared with a fixed pricing mechanism such as pre-negotiated contracts (*e.g.*, the FIT program in Ontario, Canada) and flat-rate pricing [2], auctions reduce the chance of over-pricing

and under-pricing, better match resources with buyers that value them the most, and hence increase resource utilization efficiency, system-wide social welfare, and seller revenue. Auction based solutions have recently enjoyed success in areas including online advertisement and cloud computing, and are indeed what have been practiced in wholesale electricity markets for decades, although in a much washed down version that avoids technical complexities of UCP [29].

An electricity auction outcome contains information on not only which microgrids win and at what prices, but also a generation schedule in the case microgrids sell power back to the grid, satisfying technical limitations of generators as specified in the UCP constraints. Another key challenge arises from strategic bidders, who are driven by their own commercial interests and may submit falsified bids to maximize their own utilities. Truthfully bidding, as a desirable property in practical auction mechanisms, eliminates such strategic bids from selfish bidders. A well-known truthful auction mechanism is the celebrated Vickrey-Clarke-Groves (VCG) auction, which ensures truthfulness and economical efficiency in terms of social welfare maximization. Unfortunately, the winner determination problem used in the VCG mechanism is proven NP-hard in the two electricity trading problems we study. Hence directly applying the VCG auction becomes computationally infeasible as the system size grows. Approximation algorithms, as an alternative to optimally solving NP-complete problems, have been substantially studied over the past few decades, providing near-optimal solutions within polynomial time. Unfortunately, plugging an approximate algorithm into the VCG framework may result in clearly non-truthful auction mechanisms [26].

The auction mechanism we design in this work follows a randomized auction framework, which translates approximate social welfare maximization algorithms into an approximate-truthful auction and an absolute-truthful auction respectively, with the help of a fractional VCG auction and a convex decomposition technique. The resulting auctions execute in polynomial time, are (approximately) truthful in expectation, and guarantee approximate economic efficiency. At high level, the randomized auction framework consists of three main steps: (i) a fractional VCG auction is first conducted after relaxing integer decision variables; (ii) then a carefully designed convex decomposition technique is utilized to break the fractional solution down into a convex combination of integer solutions; (iii) finally feasible integer solutions are chosen by viewing their corresponding weights as probabilities, and payments are computed for helping the randomized auction inherit truthfulness from the fractional VCG auction.

Specifically, we first study the market where the grid purchases electricity from microgrids. We design an iterative primal-dual algorithm for the winner determination problem (WDP1) by exploiting the underlying structure of WDP1 through Lagrange relaxation and problem decomposition. We then simulate a fractional VCG auction by relaxing the integer decision variables, which is truthful and computationally efficient. Next we utilize a pair of carefully designed primal and dual linear programs to decompose the fractional solution into a convex combination of mixed integer solutions by employing the approximation algorithm as a separation oracle, under UCP constraints. The decomposition process can be completed in polynomial time using the ellipsoid method, providing a series of weights for selected mixed integer solutions. Finally, each candidate mixed integer solution is chosen with probability equal to its corresponding weight. The randomized auction guarantees the same approximation ratio ρ as the plug-in ρ -approximation algorithm does, under truthful bidding. Truthfulness of the randomized auction is not absolutely guaranteed, but is provided in

a best-effort fashion. We leave the design of an absolute-truthful, computationally efficient and approximately social welfare maximizing auction for this scenario as future work.

We next study the market where microgrids purchase electricity from the grid for their supply deficiencies. An α -approximation algorithm is first designed for the detailed problem without considering truthfulness, based on greedy primal-dual algorithm design. We then simulate a fractional VCG auction based on the LP relaxation (LPR) of the social welfare maximization, WDP2. Next, we exploit the underlying packing nature of WDP2, and solve a pair of tailored primal-dual linear programs again to decompose the fractional solution to WDP2 into a combination of weighted integer solutions, using the ellipsoid algorithm with the α -approximation algorithm acting as a separation oracle. Finally, we select integer solutions using the calculated weights as corresponding probabilities. Absolute truthfulness is guaranteed by that of the fractional VCG auction. The randomized auction again guarantees the same approximation ratio α in terms of social welfare as the cooperative α -approximation algorithm does.

In the rest of the paper, we review related work in Sec. 2, and present the system model, the randomized auction framework as well as an example approximation algorithm in Sec. 3. In Sec. 4, we design another variant of the randomized auction framework and its corresponding approximation algorithm. Performance evaluation is presented in Sec. 5. Sec. 6 concludes the paper.

2. RELATED WORK

Auctions are extensively employed in traditional power grids, *e.g.*, Nicolaisen *et al.* [27] propose a computational wholesale electricity market operating in a *clearing house* double-auction manner. Similarly, Tesfatsion [29] proposes centrally administered wholesale electricity markets with congestion management using an auction approach. Neither of them takes UCP constraints into consideration. McGuire [24] propose an auction for heuristically minimizing operational cost with substantially simplified unit commitment constraints, by collecting cost bids from each generating unit. No proven guarantee on the performance or computational complexity analysis is provided.

Microgrids, as an emerging paradigm of power generation and supply, have been studied in a series of recent work. Lasseter *et al.* [19] was among the first to propose the concept of microgrids. Barnes *et al.* [6] summarize dozens of existing and undergoing demonstration projects of microgrids across America, Asia and Europe. Lu *et al.* [23] investigate microgrids, considering renewable energy (wind, solar) and co-generation, and propose an online algorithm CHASE with a small competitive ratio in operation cost. With limited prediction into the future, CHASE can be extended to behave more intelligently.

A few studies recently appeared in the literature of auction design for microgrids. For example, Dimeas *et al.* [10] present a distributed control approach for microgrids, using an auction algorithm for the solution of the symmetric assignment problem. Yet they fail to consider the key unit commitment problem in the proposed solution. Tsikalakis *et al.* [30] consider demand side bidding where consumers of the microgrids submit bids to purchase energy from microgrids. Based on such bids, the microgrid central controller chooses either to minimize operational cost or to maximize profit. Their auction is not proven to be truthful.

The celebrated VCG auction mechanism due to Vickrey [31], Clarke [9] and Groves [15], represents a general truthful auction framework, in which no rational buyers have motivation to submit falsified bids. Formally, an auction mechanism is *truthful* if bidding the true valuation is a dominant strategy for each bidder. VCG

auctions are proven to be the only type of auctions that can simultaneously guarantee truthfulness and absolute economic efficiency. A VCG auction requires optimally solving the social welfare maximization problem multiple times, for calculating allocation rules as well as payments of winning buyers. Unfortunately, the underlying optimization problem is often NP-hard, making the VCG auction computationally infeasible, especially when facing a large number of bidders. Approximation algorithms are known to be an efficient alternative for solving NP-hard problems, computing sub-optimal solutions in polynomial time. Unfortunately a VCG auction loses its truthfulness when one applies an approximation algorithm instead of an optimal algorithm for social welfare maximization [26].

In a sequence of recent work that initiated from theoretical computer science [8, 20, 25, 32], a polynomial-time convex decomposition technique is designed for converting fractional solution for an NP-hard problem, modelled as a linear integer program, into a weighted combination of integer solutions. Such a decomposition technique enables a randomized auction framework that automatically translates a centralized cooperative approximation algorithm into an auction mechanism, achieving the same social welfare approximation ratio as the plug-in approximation algorithm does, while guaranteeing truthful bidding. A key property exploited in the decomposition is the packing or covering property. A linear program is a *packing* LP if it is of the form: Maximize $\mathbf{b}^T \mathbf{y}$, subject to: $\mathbf{A}^T \mathbf{y} \leq \mathbf{c}$, $\mathbf{y} \geq \mathbf{0}$, where the matrix \mathbf{A} and vectors \mathbf{b} and \mathbf{c} are non-negative. The dual of a packing LP is a covering LP.

Dughmi and Roughgarden [11] recently proposed another technique that transfers an approximation algorithm into a truthful auction mechanism with the same social welfare approximation ratio. This approach requires that the social welfare maximization problem admits an Full Polynomial-Time Approximation Scheme (FPTAS), which is a stronger requirement than having a polynomial-time constant-ratio approximation algorithm. Many NP-hard optimization problems (including all that are proven APX-hard) do not have FPTAS.

3. THE MICROGRID-TO-GRID ELECTRICITY MARKET

3.1 Generator constraints and social welfare optimization

We consider the microgrid-to-grid market in this section, consisting of a regional power grid and a set \mathcal{N} ($|\mathcal{N}| = n$) of microgrids. Each microgrid has its own generator that produces electrical power. An example microgrid in operation is shown in Fig. 2. Each generator incurs an operational cost in the active generation mode, which depends on the type of generation (*e.g.*, gas or diesel). There is further an amortized infrastructure and maintenance cost. Both costs are private information at the microgrid. The regional power grid (the *auctioneer*) solicits electricity sales from the microgrids (the *bidders*) during periods of under-supply, through a (reversed) auction. If the total amount of energy purchased from the auction falls short to bridge the gap, the grid may further purchase electricity from generation plants based in the near-term markets (which are usually more expensive) [27], or from electricity storage [12]. We consider an operation period \mathcal{T} from $t = 1$ to $t = T$, during which the demand of the grid can be predicted. In practice, auctions in an electricity market are conducted based on forecasts of power demand in an upcoming time period (*e.g.*, the next 24 hours or several days [3]). Correspondingly, near-future demands are assumed to be known in advance.



Figure 2: The Sendai microgrid [1], with an output capacity of 1 MW, remained in full service during and after the March 2011 Great East Japan Earthquake, supplying power and heat to the community around Tohoku Fukushi University.

Let $x_i(t)$ be a binary variable indicating whether microgrid $i \in \mathcal{N}$ is ON or OFF at $t \in \mathcal{T}$. $y_i(t)$ indicates the power output of microgrid $i \in \mathcal{N}$. $z(t)$ is the total energy supplied by plants or batteries rather than microgrids at $t \in \mathcal{T}$. $s_i(t)$ is the amortized infrastructure/maintenance cost of microgrid $i \in \mathcal{N}$ at time t . $u_i(t)$ is a unit cost (\$ per MWh) of microgrid $i \in \mathcal{N}$ at $t \in \mathcal{T}$. $p(t)$ is a unit cost (\$ per MWh) of the energy supplied by plants or batteries rather than microgrids at $t \in \mathcal{T}$. $D(t)$ is the power demand at $t \in \mathcal{T}$, known by the grid. δ is the maximum output level change between two consecutive time slots. Δ^+ is the maximum output level at the first time slot of a commitment period, known as startup ramp limit. Similarly Δ^- is the maximum output level at the last time slot of a commitment period, known as shutdown ramp limit.

The auction of electricity from the microgrids to the regional power grid is conducted once, at $t = 1$, for the period $\mathcal{T} = [1, T]$, based on predicted information for \mathcal{T} . The grid (auctioneer) schedules its power supply for $t \in \mathcal{T}$. Each microgrid (bidder) i submits its bid containing its private costs $s_i(t)$ and $u_i(t)$, and can contribute a maximum output level of P_{max} . At each time t , total supply should cover total demand in the grid:

$$\sum_i y_i(t) + z(t) \geq D(t), \forall t \in \mathcal{T}$$

Then we have the UCP constraints dictating generator schedules. We assume that all microgrids are off at $t = 0$, and all microgrids keep the same status as $t = T$ after the operation period \mathcal{T} , i.e., $x_i(T) = x_i(T+1) = x_i(T+2) = \dots$, for facilitating constraint formulation.

First, a generator, once turned ON, must remain active for at least T_{on} time slots. Similarly it must stay deactive for at least T_{off} time slots once turned OFF:

$$UCP1: \begin{cases} \sum_{\tau=t+1}^{t+T_{on}} x_i(\tau) \geq T_{on}(x_i(t+1) - x_i(t)), \forall t \in \mathcal{T}, i \in \mathcal{N} \\ \sum_{\tau=t+1}^{t+T_{off}} (1 - x_i(\tau)) \geq T_{off}(x_i(t) - x_i(t+1)), \forall t \in \mathcal{T}, i \in \mathcal{N} \end{cases}$$

The output level of a microgrid cannot exceed its capacity P_{max} , and the minimal output is P_{min} if it is active:

$$UCP2: \begin{cases} P_{max}x_i(t) \geq y_i(t), \forall t \in \mathcal{T}, i \in \mathcal{N} \\ y_i(t) \geq P_{min}x_i(t), \forall t \in \mathcal{T}, i \in \mathcal{N} \end{cases}$$

Furthermore, the output level of a generator cannot vary abruptly. The *ramping rate* is upper-bounded:

$$UCP3: \begin{cases} y_i(t) - y_i(t-1) \leq \delta x_i(t-1) \\ + \Delta^+(1 - x_i(t-1)), \forall t \in \mathcal{T}, i \in \mathcal{N} \end{cases}$$

$$y_i(t-1) - y_i(t) \leq \delta x_i(t) + \Delta^-(1 - x_i(t)), \forall t \in \mathcal{T}, i \in \mathcal{N}$$

There are three types of costs. $\sum_{i,t} s_i(t)x_i(t)$ is the amortized hardware/maintenance cost¹; $\sum_{i,t} u_i(t)y_i(t)$ is fuel cost; $\sum_t p(t)z(t)$ is the total expense in power purchase from plants/batteries, which is a cost of the regional power grid (auctioneer). A system-wide (grid and microgrids) social welfare maximization translates into the objective of minimizing the aggregated cost, while meeting the grid's demand. The winner determination problem (WDP1) with $x_i(t)$, $y_i(t)$ and $z(t)$ as decision variables is:

$$\text{minimize } \sum_{i,t} s_i(t)x_i(t) + \sum_{i,t} u_i(t)y_i(t) + \sum_t p(t)z(t) \quad (1)$$

subject to:

$$\sum_i y_i(t) + z(t) \geq D(t), \quad \forall t \quad (1a)$$

$$\sum_{\tau=t+1}^{t+T_{on}} x_i(\tau) \geq T_{on}(x_i(t+1) - x_i(t)), \quad \forall t, i \quad (1b)$$

$$\sum_{\tau=t+1}^{t+T_{off}} (1 - x_i(\tau)) \geq T_{off}(x_i(t) - x_i(t+1)), \quad \forall t, i \quad (1c)$$

$$P_{max}x_i(t) \geq y_i(t), \quad \forall t, i \quad (1d)$$

$$y_i(t) \geq P_{min}x_i(t), \quad \forall t, i \quad (1e)$$

$$y_i(t) - y_i(t-1) \leq \delta x_i(t-1) + \Delta^+(1 - x_i(t-1)), \quad \forall t, i \quad (1f)$$

$$y_i(t-1) - y_i(t) \leq \delta x_i(t) + \Delta^-(1 - x_i(t)), \quad \forall t, i \quad (1g)$$

$$x_i(t) \in \{0, 1\}, y_i(t) \geq 0, z(t) \geq 0, \quad \forall t, i \quad (1h)$$

Generator scheduling under UCP is in general NP-hard [16]. This rules out a direct application of the VCG auction mechanism for truthful auction design. We instead exploit the underlying structure of WDP1 for designing a randomized auction that is computationally efficient.

3.2 The randomized auction framework

The electricity auction we design leverages a randomized auction framework that consists of three main steps, as illustrated in Algorithm 1.

Step 1. Simulating the fractional VCG auction. We first simulate the fractional VCG auction with the linear programming relaxation (LPR) of WDP1 as the aggregated cost minimization problem, which is obtained by relaxing the binary variable $x_i(t)$ to $0 \leq x_i(t) \leq 1$. Such a fractional VCG auction is truthful but the solution is fractional and therefore infeasible. Let (x^*, y^*, z^*) be the optimal fractional solution to the LPR of WDP1. Prices charged in the fractional VCG mechanism, Π^f , are computed as follows:

$$\Pi_k^f = \sum_{i,t} (s_i(t)x_i'(t) + u_i(t)y_i'(t) + p(t)z'(t)) - \sum_{i \neq k, t} (s_i(t)x_i^*(t) + u_i(t)y_i^*(t) + p(t)z^*(t)) \quad (2)$$

¹There are two types of infrastructure/maintenance costs: amortized hardware cost and running cost. The former is amortized over the generator's lifetime, while the latter is applicable only when the generator is turned on. This work focuses on the latter.

Algorithm 1 A Randomized Microgrid-to-Grid Power Auction

- 1: **Step 1. Simulating the fractional VCG auction.**
 - 2: — Compute the fractional VCG allocation \mathbf{x}^* , and payment Π^f , through solving the LPR of WDP1.
 - 3: **Step 2. Decomposing the optimal fractional solution**
 - 4: — Decompose the fractional solution $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{z}^*)$ to a convex combination of mixed integer solutions, *i.e.*, $\sum_{q \in \mathcal{I}} \lambda^q (\mathbf{x}^q, \mathbf{y}^q, \mathbf{z}^q) \leq \rho (\mathbf{x}^*, \mathbf{y}^*, \mathbf{z}^*)$, through solving a pair of primal-dual LPs in (4) and (5) using the ellipsoid method, leveraging a ρ -approximation algorithm verifying the integrality gap of ρ as a separation oracle.
 - 5: **Step 3. Randomized winner selection and payment**
 - 6: — Select each $(\mathbf{x}^q, \mathbf{y}^q, \mathbf{z}^q)$ randomly with probability λ^q .
 - for each winning microgrid i : charge a payment $\Pi_i = \Pi_i^f \frac{\sum_t (s_i(t)x_i^q(t) + u_i(t)y_i^q(t))}{\sum_t (s_i(t)x_i^*(t) + u_i(t)y_i^*(t))}$, if $\sum_t (s_i(t)x_i^*(t) + u_i(t)y_i^*(t)) \neq 0$; $\Pi_i = 0$ otherwise.
-

where $(\mathbf{x}', \mathbf{y}', \mathbf{z}')$ is an optimal fractional solution without buying any power from microgrid $k \in \mathcal{N}$.

Step 2. Decomposing the optimal fractional solution.

Applying the recent convex decomposition technique [8, 20, 25], we decompose the optimal fractional solution into a convex combination of integral solutions each with a fractional weight that sums up to 1. This step requires a *separation oracle*, an effective polynomial-time approximation algorithm to WDP1 satisfying:

$$\sum_{i,t} s_i(t)x_i(t) + \sum_{i,t} u_i(t)y_i(t) + \sum_t p(t)z(t) \leq \rho OPT_{LPR1} \quad (3)$$

where OPT_{LPR1} is the value of the objective function for the optimal fractional solution $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{z}^*)$ for WDP1.

The goal of the decomposition is to find combination weights $\lambda^q \geq 0$, such that $\sum_{q \in \mathcal{I}} \lambda^q = 1$, $\sum_{q \in \mathcal{I}} \lambda^q \mathbf{x}^q \leq \rho \mathbf{x}^*$, $\sum_{q \in \mathcal{I}} \lambda^q \mathbf{y}^q \leq \rho \mathbf{y}^*$ and $\sum_{q \in \mathcal{I}} \lambda^q \mathbf{z}^q \leq \rho \mathbf{z}^*$, where \mathcal{I} is the index set for all feasible mixed integer solutions to WDP1.

We compute such a weight vector λ through solving the following pair of primal and dual LPs:

$$\text{Primal: maximize } \sum_{q \in \mathcal{I}} \lambda^q \quad (4)$$

subject to:

$$\sum_{q \in \mathcal{I}} \lambda^q (\mathbf{x}^q, \mathbf{y}^q, \mathbf{z}^q) \leq \rho (\mathbf{x}^*, \mathbf{y}^*, \mathbf{z}^*) \quad (4a)$$

$$\sum_{q \in \mathcal{I}} \lambda^q \leq 1 \quad (4b)$$

$$\lambda^q \geq 0, \forall q \in \mathcal{I} \quad (4c)$$

$$\text{Dual: minimize } \rho \left(\sum_{i,t} \alpha_i(t)x_i^*(t) + \sum_{i,t} \beta_i(t)y_i^*(t) + \sum_t \eta(t)z^*(t) \right) + \gamma \quad (5)$$

subject to:

$$\sum_{i,t} \alpha_i(t)x_i^q(t) + \sum_{i,t} \beta_i(t)y_i^q(t) + \sum_t \eta(t)z^q(t) \geq 1 - \gamma, \forall q \in \mathcal{I} \quad (5a)$$

$$\alpha, \beta, \eta \geq 0, \gamma \geq 0 \quad (5b)$$

THEOREM 1. *LPs (4) and (5) can be solved in polynomial time, and the optimal value is 1.*

PROOF. First $(\alpha = 0, \beta = 0, \eta = 0, \gamma = 1)$ is a feasible solution to the dual, hence the optimal value is at most 1. By way of contradiction, we assume that

$$\rho \left(\sum_{i,t} \alpha_i(t)x_i^*(t) + \sum_{i,t} \beta_i(t)y_i^*(t) + \sum_t \eta(t)z^*(t) \right) + \gamma < 1$$

To solve the dual, we need to apply a cooperative approximation algorithm to WDP1:

$$\begin{aligned} & \sum_{i,t} \alpha_i(t)x_i^q(t) + \sum_{i,t} \beta_i(t)y_i^q(t) \\ & + \sum_t \eta(t)z(t) \leq \rho OPT_{LPR1}(\alpha, \beta, \eta) \end{aligned}$$

where $(\mathbf{x}^q, \mathbf{y}^q, \mathbf{z}^q)$ and $OPT_{LPR1}(\alpha, \beta, \eta)$ are a feasible mixed integer solution and the value of the optimal fractional solution, respectively, to WDP1 with α as bidding prices, β as fuel prices and η as the unit prices of energy purchased from plants or batteries. Note that:

$$\begin{aligned} OPT_{LPR1}(\alpha, \beta, \eta) & \leq \sum_{i,t} \alpha_i(t)x_i^*(t) \\ & + \sum_{i,t} \beta_i(t)y_i^*(t) + \sum_t \eta(t)z^*(t) \end{aligned}$$

which implies

$$\begin{aligned} & \sum_{i,t} \alpha_i(t)x_i^q(t) + \sum_{i,t} \beta_i(t)y_i^q(t) + \sum_t \eta(t)z^q(t) \\ & \leq \rho \left(\sum_{i,t} \alpha_i(t)x_i^*(t) + \sum_{i,t} \beta_i(t)y_i^*(t) + \sum_t \eta(t)z^*(t) \right) \\ & < 1 - \gamma \end{aligned}$$

The above inequality violates the dual constraints, leading to a contradiction. Therefore the optimal dual objective value is 1. Following strong LP duality, we conclude that the optimal primal objective value is 1 as well.

We observe that the primal LP (4) has an exponential number of variables, which may take exponential time to solve directly. We instead resort to the dual LP (5) that has an exponential number of constraints. The ellipsoid method can solve the problem within polynomial time despite an exponential number of constraints [14]. The cooperative approximation algorithm serves as a separation oracle [8], providing a polynomial number of hyperplanes to cut the ellipsoid, making the dual LP solvable in polynomial time. Each hyperplane corresponds to a constraint in the dual, providing a feasible solution $(\mathbf{x}^q, \mathbf{y}^q, \mathbf{z}^q)$ as well as corresponding primal variable λ^q . The primal LP then can be transformed to an optimization problem with a polynomial number of variables corresponding to these hyperplanes. We hence can solve the primal LP in polynomial time, obtaining weights of the convex decomposition that sum to 1. \square

Step 3. Randomized winner selection and payment. Following the decomposition, each possible solution $(\mathbf{x}^q, \mathbf{y}^q, \mathbf{z}^q)$ is selected randomly with a probability equal to its corresponding weight λ^q

computed in the decomposition in the second step. Then microgrid i receives a payment $\Pi_i = \Pi_i^f \frac{\sum_t (s_i(t)x_i^q(t) + u_i(t)y_i^q(t))}{\sum_t (s_i(t)x_i^*(t) + u_i(t)y_i^*(t))}$, if $\sum_t (s_i(t)x_i^*(t) + u_i(t)y_i^*(t)) \neq 0$; $\Pi_i = 0$ otherwise.

Assume truthful bidding, and then the expected social welfare is:

$$\begin{aligned} & \sum_q (\lambda^q \mathbf{s}^T \mathbf{x}^q + \lambda^q \mathbf{u}^T \mathbf{y}^q + \lambda^q \mathbf{p}^T \mathbf{z}) \\ & \leq \rho \mathbf{s}^T \mathbf{x}^* + \rho \mathbf{u}^T \mathbf{y}^* + \rho \mathbf{p}^T \mathbf{z}^* \leq \rho OPT_{WDP1} \end{aligned}$$

where OPT_{WDP1} is the optimal mixed integer solution. Above inequality implies that the randomized auction framework achieves an approximation ratio of ρ with respect to the aggregated cost if users bid truthfully.

Recall that the auction in Sec. 3 is a reverse auction. Assume truthful bidding again, and then we have the expected utility of microgrid $i \in \mathcal{N}$:

$$\begin{aligned} & E[\Pi_i] - \bar{v}_i \left(\sum_q \lambda^q (\mathbf{x}^q, \mathbf{y}^q) \right) = \\ & \left(\Pi_i^f - \sum_t (s_i(t)x_i^*(t) + u_i(t)y_i^*(t)) \right) \frac{\sum_{t,q} \lambda^q (s_i(t)x_i^q(t) + u_i(t)y_i^q(t))}{\sum_t (s_i(t)x_i^*(t) + u_i(t)y_i^*(t))} \end{aligned}$$

Therefore the expected utility of microgrid i is larger than zero since the fractional VCG auction is individual rational, *i.e.*, $\Pi_i^f - \sum_t (s_i(t)x_i^*(t) + u_i(t)y_i^*(t)) \geq 0$.

3.3 The approximation algorithm for WDP1

Plug-in nature of the approximation algorithm. The randomized auction framework in Algorithm 1 requires an efficient approximation algorithm for solving WDP1. We emphasize that any approximation algorithm to WDP1 can be applied as a plug-in module to the auction framework. Such a plug-in approximation algorithm should be computationally efficient, and computes a feasible solution with a cost that is as close as possible to the LPR of WDP1. Below we present an example approximation algorithm design based on primal-dual optimization.

An example primal-dual algorithm. A key observation regarding WDP1 is that (1a) is the only set of constraints that couple schedule variables from different microgrids. Lagrangian relaxation can naturally be applied to relax this set of constraints, such that the relaxed optimization problem is decomposable into a series of independent single-microgrid UCP optimization. We introduce a non-negative Lagrangian multiplier ξ_t corresponding to each coupling constraint (1a), remove constraint (1a), and add a corresponding penalty term into the objective function of WDP1, which becomes:

$$\begin{aligned} & \sum_{i,t} s_i(t)x_i(t) + \sum_{i,t} y_i(t)u_i(t) + \sum_t z(t)p(t) \\ & + \sum_t \xi_t (D(t) - \sum_i y_i(t) - z(t)) \\ & = \sum_{i,t} s_i(t)x_i(t) + \sum_{i,t} y_i(t)(u_i(t) - \xi_t) \\ & + \sum_t z(t)(p(t) - \xi_t) + \sum_t \xi_t D(t) \end{aligned}$$

Note that the objective function may be unbounded when $p(t) < \xi_t$, $z(t) \rightarrow +\infty$. Hence primal feasibility requires that $p(t) \geq \xi_t, \forall t$. The Lagrangian dual problem to WDP1 is then:

$$\text{maximize } \mathcal{L}(\boldsymbol{\xi}) \quad (6)$$

subject to

$$\xi_t \leq p(t), \quad \forall t \in \mathcal{T} \quad (6a)$$

$$\boldsymbol{\xi} \succeq 0 \quad (6b)$$

where $\mathcal{L}(\boldsymbol{\xi}) = \min_{\mathbf{x}, \mathbf{y} \in \mathcal{P}} \left(\sum_{i,t} s_i(t)x_i(t) + \sum_{i,t} y_i(t)(u_i(t) - \xi_t) + \sum_t \xi_t D(t) \right)$, and \mathcal{P} is the polytope defined by constraints (1b) – (1g).

Next, the Lagrangian dual problem can be separated into a series of single-microgrid optimal schedule problems, solved for each microgrid independently. We can write $\mathcal{L}(\boldsymbol{\xi}) = \sum_i g_i(\boldsymbol{\xi}) + \sum_t \xi_t D(t)$, where $g_i(\boldsymbol{\xi}), \forall i \in \mathcal{N}$ are defined as follows:

$$g_i(\boldsymbol{\xi}) = \min_{\mathcal{P}} \left(\sum_t s_i(t)x_i(t) + \sum_t y_i(t)(u_i(t) - \xi_t) \right)$$

The Lagrange dual problem can be solved by a subgradient algorithm, as shown in Algorithm 2. In each iteration, the sub-problems for each microgrid are first solved, then we update the dual variables by $\xi_t = \xi_t + b_k(D(t) - \sum_i y_i(t) - z(t))$. Due to constraint (6a), the inner optimization $\mathcal{L}(\boldsymbol{\xi})$ always chooses $z(t) = 0$, and we ignore $z(t)$. Here b_k is a step size sequence that satisfies (i) $\lim_{k \rightarrow \infty} b_k = 0$, and (ii) $\sum_{k=0}^{\infty} b_k = \infty$. A typical such sequence is in the form of $3/(2k+1)$.

Algorithm 2 A subgradient algorithm for WDP1

- 1: **Dual initialization.** Set $\xi_t = p(t), \forall t \in \mathcal{T}$
 - 2: **repeat**
 - 3: — **Primal update.** For each microgrid i , given current $\boldsymbol{\xi}$, compute optimal schedule $x_i(t)$ and $y_i(t)$ by solving $g_i(\boldsymbol{\xi}), \forall i$, through dynamic programming.
 - 4: — **Dual update.** Given current \mathbf{x} and \mathbf{y} , update the dual variables by $\xi_t = \xi_t + b_k(D(t) - \sum_i y_i(t))$.
 - 5: **until** convergence
-

The single-microgrid optimization is a classic UCP problem with one generator, and can be solved by dynamic programming efficiently [13, 28].

Upon convergence of the subgradient algorithm, the relaxed primal constraint (1a) on grid-wide demand-supply is not always satisfied, although it has been observed that the gaps are rather small [16]. In this study, primal feasibility can be ensured by: (1) purchasing more from power reservoirs (plants or batteries), *i.e.*, increasing $z(t)$ in WDP1 to satisfy the demand; or using existing techniques from the literature, *e.g.*, Frangioni *et al.* [13] use a heuristic strategy to adjust the commitment status of generators to meet the demand; Aoki *et al.* [5] solve least square type problems to find a feasible solution.

The approximation ratio ρ . The auction framework in Algorithm 1 requires the approximation algorithm to compute an integral solution that is within an integrality gap of ρ from the LPR. While the subgradient method can always compute optimal solutions for the dual (and hence the primal) when the original optimization problem is a linear program, it leads to a duality gap when the original problem is a linear integer program. However, fortunately, a series of previous studies have verified that such a duality gap is extremely small for UCP-type problems, often within 1-2% [16]. The overall ratio ρ should be bounded by a constant in most cases, and can be estimated by empirically studies. The auction framework actually allows some flexibility in the value selection of ρ — precisely selecting the smallest upper-bound for the integrality gap serves

the best interest of approximate truthfulness and social welfare but is hard; choosing a non-exact upper-bound works in practice with some compromise in truthfulness and social welfare. Our auction in this section is not an absolutely truthful auction. Nonetheless, it strikes to achieve a high level of (approximate) truthfulness, and from the bidder's point of view, it is unclear what strategy other than truthful bidding will lead to a higher utility.

4. THE GRID-TO-MICROGRID ELECTRICITY MARKET

4.1 Market constraints and social welfare maximization

In this section, we consider a regional power grid (auctioneer) who sells electricity to a number of microgrids (bidders) denoted by \mathcal{M} . The system runs in a round-by-round fashion from $t = 1$ to $t = T$, for which time period a microgrid $i \in \mathcal{M}$ submits one or more bids. B_i denotes all bids submitted by microgrid $i \in \mathcal{M}$. Each bid $j \in B_i$ of microgrid i , $(b_{i,j}, d_{i,j})$, specifies a willingness to pay, $b_{i,j}$, for a demand curve $d_{i,j}$ that contains entries $d_{i,j}(t)$, which is the amount of electricity demand at each time slot t . In practice, $d_{i,j}$ is derived from a microgrid's prediction of near future power consumption. Similar to the microgrid-to-grid market in Sec. 3, the auction of electricity from the regional power grid to the microgrids is conducted at $t = 1$, when all bids are collected. Moreover, $\chi_{i,j}$ is a binary decision variable indicating whether microgrid i wins bid j or not. Let $\bar{v}_i(\chi)$ denote the true valuation of microgrid i , known by microgrid i only. We assume that $\bar{v}_i(\chi)$ is linear in χ . The available capacity of the regional grid, $C(t)$, varies over time, due to fluctuation in both the output from wind and solar generators and background demand. For example, Fig. 3 illustrates the fluctuation of hourly demand and available capacity within Ontario, Canada [3]. We assume that at each time, a single bid cannot exceed the total available capacity of the grid, i.e., $C(t) > R(t) \triangleq \max_{i,j} d_{i,j}(t)$.

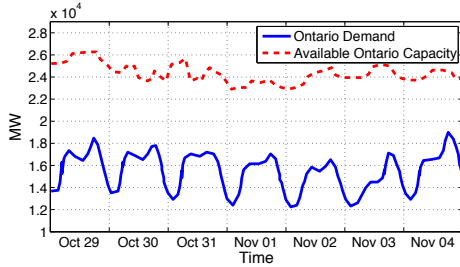


Figure 3: The fluctuation of hourly Electrical power demand and available capacity from October 29 to November 04, 2013 within Ontario, Canada.

We employ the XOR-bidding language [22], such that among all bids submitted by a microgrid, at most one bid can win:

$$\sum_j \chi_{i,j} \leq 1, \forall i \in \mathcal{M}$$

Furthermore, the total power demand of all winning bids at each time slot cannot exceed the available capacity of the grid:

$$\sum_{i,j} d_{i,j}(t) \chi_{i,j} \leq C(t), \forall t$$

The winner determination problem (WDP2) then maximizes social welfare (total utility of both the grid and the microgrids, with payments from the latter cancelling revenue of the former), under

the above two constraints:

$$\text{maximize } \sum_{i,j} b_{i,j} \chi_{i,j} \quad (7)$$

subject to:

$$\sum_j \chi_{i,j} \leq 1, \quad \forall i \in \mathcal{M} \quad (7a)$$

$$\sum_{i,j} d_{i,j}(t) \chi_{i,j} \leq C(t), \quad \forall t \quad (7b)$$

$$\chi_{i,j} \in \{0, 1\}, \quad \forall i \in \mathcal{M}, j \in B_i \quad (7c)$$

A salient feature in the grid-to-microgrid electricity market lies in the temporal dimension constraint that couples demand-supply across different time slots, as evident in (7b). As a result, WDP2 becomes NP-hard (proven below), making it highly non-trivial to design an auction that simultaneously guarantees truthfulness and good social welfare efficiency.

THEOREM 2. *WDP2 in (7) is NP-hard.*

PROOF. We present a polynomial-time reduction from the knapsack problem, a classic NP-hard problem [18] defined as:

$$\max_{\chi} \left\{ \sum_{i=1}^n v_i \chi_i \text{ s.t. } \sum_{i=1}^n w_i \chi_i \leq W, \chi_i \in \{0, 1\} \right\}$$

Let us consider a special case of the WDP2: $|\mathcal{M}| = n$, each bidder submits a single bid and only one time slot is considered. WDP2 degrades into:

$$\max_{\chi} \left\{ \sum_i b_i \chi_i \text{ s.t. } \sum_i d_i \chi_i \leq C, \chi_i \in \{0, 1\}, \forall i \in \mathcal{M} \right\}$$

which is exactly a classic knapsack problem. Apparently, the reduction can be done within polynomial time. Therefore, WDP2 in (7) is NP-hard. \square

4.2 The randomized auction framework

Theorem 2 suggests that solving WDP2 becomes daunting as the number of microgrids grows, which rules out a direct application of the VCG auction mechanism in the grid-to-microgrid market, since it requires optimally solving multiple WDP2 instances. We resort to the LPR of WDP2 by relaxing constraint (7c) to $(\chi_{i,j} \leq 1, \forall i \in \mathcal{M}, j \in B_i)$, is redundant and hence ignored):

$$\chi_{i,j} \geq 0, \forall i \in \mathcal{M}, j \in B_i \quad (7c')$$

Similar to the framework applied in Sec. 3, we design an electricity auction following the randomized auction framework below.

Step 1. Simulate the fractional VCG auction. Compute the optimal fractional solution χ^* to the LPR of WDP2. Compute fractional VCG payments: buyer k pays $\pi_k^f = \sum_{i,j} b_{i,j} \chi_{i,j}^* - \sum_{i \neq k, j} b_{i,j} \chi_{i,j}^*$, where χ^* is the optimal solution to WDP2's LPR without selling any energy to microgrid k . The fractional (infeasible) solution χ^* is decomposed into integral (feasible) solutions in Step 2.

Step 2. Decompose the fractional solution χ^* . We next decompose χ^* into a combination of a series of feasible integer solutions by finding non-negative multipliers $\{\nu_q\}_{q \in \mathcal{J}}$, such that $\sum_{q \in \mathcal{J}} \nu_q = 1$, $\sum_{q \in \mathcal{J}} \nu_q \chi^q = \chi^*/a$, where \mathcal{J} is the index set for all feasible integer solutions to WDP2.

Similar to the decomposition in Sec. 3, the decomposition step solves a pair of primal-dual LPs through the ellipsoid method, which requires an approximation algorithm to WDP2 that computes an integer solution χ , satisfying:

$$\sum_{i,j} b_{i,j} \chi_{i,j} \geq 1/a \sum_{i,j} b_{i,j} \chi_{i,j}^*$$

Step 3. Randomized winner selection and payment scaling. Select χ^q with probability ν^q , with following defined scale-down payment π_i for guaranteeing truthfulness and individual rationality, as analyzed below.

$$\pi_i = \begin{cases} \pi_i^f \frac{\sum_j b_{i,j} \chi_{i,j}^q}{\sum_j b_{i,j} \chi_{i,j}^*} & \text{if } \sum_j b_{i,j} \chi_{i,j}^* \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Truthfulness. Since the auction in Sec. 4 is an ordinary (forward) auction, the expected utility of microgrid i is:

$$\bar{v}_i(\sum_q \nu^q \chi_j^q(t)) - E[\pi_i] = (\bar{v}_i(\chi^*) - \pi_i^f)/a.$$

Then truthfulness of the 3-step auction follows from the truthfulness of the fractional VCG auction.

The convex decomposition. We compute values for ν^q , which are weights required in the decomposition of χ^* for solution feasibility, by solving a pair of primal-dual LPs below:

$$\text{Primal: minimize } \sum_{q \in \mathcal{J}} \nu^q \quad (8)$$

subject to:

$$\begin{aligned} \sum_{q \in \mathcal{J}} \nu^q \chi^q &= \chi^*/a \\ \sum_{q \in \mathcal{J}} \nu^q &\geq 1 \\ \nu^q &\geq 0, \forall q \in \mathcal{J} \end{aligned}$$

$$\text{Dual: maximize } \sum_{i,j} w_{i,j} \chi_{i,j}^*/a + \iota \quad (9)$$

subject to:

$$\begin{aligned} \sum_{i,j} w_{i,j} \chi_{i,j}^q + \iota &\leq 1, \forall q \in \mathcal{J} \\ w_{i,j} &\text{ unconstrained}, \iota \geq 0, \forall i, j \end{aligned}$$

Our goal is to solve the primal to optimal with $\sum_q \nu^q = 1$. We next prove that $\sum_q \nu^q = 1$ indeed happens at optimum.

THEOREM 3. *Given a polynomial time approximation algorithm to WDP2 that computes an integer solution χ , satisfying $\sum_{i,j} b_{i,j} \chi_{i,j} \geq 1/a \sum_{i,j} b_{i,j} \chi_{i,j}^*$, the primal (8) and the dual (9) can be optimally solved in polynomial time, with optimal objective value of 1.*

PROOF. First note that $w = 0, \iota = 1$ constitutes a feasible solution to the dual, yielding an objective value of 1. Hence the optimal dual objective is at least 1.

By way of contradiction, we assume that $\sum_{i,j} w_{i,j} \chi_{i,j}^*/a + \iota > 1$. The only potential problem is that the dual variable $w_{i,j}$ can be negative, making the approximation algorithm not applicable. Let $w_{i,j}^+ = \max\{w_{i,j}, 0\}$. Instead of $w_{i,j}$, we use $w_{i,j}^+$ as input for the approximation algorithm. Assume given $w_{i,j}^+$, we apply the approximation algorithm for verifying the integrality gap of WDP2:

$\exists q, \sum_{i,j} w_{i,j}^+ \chi_{i,j}^q \geq \sum_{i,j} w_{i,j}^+ \chi_{i,j}^{**}/a$, where χ^{**} is the optimal fractional solution given w^+ as input. Let

$$\bar{\chi}_{i,j} = \begin{cases} \chi_{i,j}^q & \text{if } w_{i,j} \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$\bar{\chi}_{i,j}$ can be verified to be a feasible integer solution to WDP2. We then have

$$\begin{aligned} \sum_{i,j} w_{i,j} \bar{\chi}_{i,j} &= \sum_{i,j} w_{i,j}^+ \chi_{i,j}^q \geq \sum_{i,j} w_{i,j}^+ \chi_{i,j}^{**}/a \\ &\geq \sum_{i,j} w_{i,j}^+ \chi_{i,j}^*/a \geq \sum_{i,j} w_{i,j} \chi_{i,j}^*/a \\ &> 1 - \iota \end{aligned}$$

implying $\sum_{i,j} w_{i,j} \bar{\chi}_{i,j} + \iota > 1$, which violates the first dual constraint. A contradiction occurs. Hence the optimal dual objective is 1. Due to strong LP duality, the optimal primal objective is 1 as well.

Similar to Sec. 3, the primal LP (8) has an exponential number of variables. We resort to the dual LP (9), which has an exponential number of constraints, and can be solved using the ellipsoid method in polynomial time, given a separation oracle (the approximation algorithm in Sec. 4.3). Once the dual LP is solved, by using a polynomial number of hyperplanes found by the separation oracle, we then convert the primal LP to an optimization with a polynomial number of variables corresponding to these hyperplanes, which can be solved in polynomial time as well. The weights then are found as a byproduct of solving the primal LP. \square

4.3 The approximation algorithm for WDP2

Dual of WDP2's LPR. We now focus on designing an approximation algorithm to WDP2 that verifies its integrality gap, as required in the ellipsoid method to solve dual (9). Introducing dual variables ϕ and ψ for constraints (7a) and (7b), respectively, we obtain the dual of WDP2's LPR:

$$\text{minimize } \sum_i \phi_i + \sum_t C(t) \psi_t \quad (10)$$

subject to:

$$\phi_i + \sum_t d_{i,j}(t) \psi_t \geq b_{i,j}, \quad \forall i, j \quad (10a)$$

$$\phi_i, \psi_t \geq 0 \quad \forall i, t \quad (10b)$$

A Greedy Primal-Dual Algorithm. We apply the classic greedy primal-dual framework that is proven effective for approximating NP-hard problems with an IP formulation [7], and design Algorithm 3 for WDP2 in (7). Algorithm 3 first initializes the primal and dual variables (lines 2-6). Then a `while` loop iteratively updates the primal and dual variables by choosing the highest unit-weight bids from microgrids (line 10-13). The `while` loop terminates when all microgrids are satisfied or the generated primal variable χ becomes infeasible, *i.e.*, allocated resources exceed the capacity of the grid. In the following analysis, let ϕ_i^τ, ψ_t^τ be the dual variables, and p_τ be the primal objective at the end of the τ -th iteration of the `while` loop.

Primal feasibility. We study the feasibility of solutions returned by Algorithm 3 first.

THEOREM 4. *Algorithm 3 provides a feasible solution to WDP2, *i.e.*, IP (7).*

PROOF. Constraint (7a) will be respected because once Algorithm 3 finds a power supply curve for user i , no more power re-

Algorithm 3 The Greedy Primal-Dual Approximation Algorithm

```

1: // Initialization
2:  $\theta = \min_t C(t)/R(t)$ ;
3:  $p = 0$ ;  $\mathcal{U} = \emptyset$ ;
4:  $\forall i, j : \chi_{i,j} = 0$ ;
5:  $\forall i : \phi_i = 0$ ;
6:  $\forall t : \psi_t = 1/C(t)$ ;
7:
8: // Iterative update of primal and dual variables:
9: while  $\sum_t C(t)\psi_t < T \exp(\theta - 1)$  AND  $\mathcal{U} \neq \mathcal{M}$  do
10:   for all  $i \in \mathcal{M} \setminus \mathcal{U}$  do
11:      $j_i = \arg \max_j \{b_{i,j}\}$ ;
12:   end for
13:    $\mu = \arg \max_{i \in \mathcal{M} \setminus \mathcal{U}} \left\{ \frac{b_{i,j_i}}{\sum_t d_{i,j_i}(t)\psi_t} \right\}$ ;
14:    $\chi_{\mu,j_\mu} = 1$ ;  $\phi_\mu = b_{\mu,j_\mu}$ ;
15:    $p = p + b_{\mu,j_\mu}$ ;  $\mathcal{U} = \mathcal{U} \cup \{\mu\}$ ;
16:   for all  $1 \leq t \leq T$  do
17:      $\psi_t = \psi_t \cdot (T \exp(\theta - 1))^{(d_{\mu,j_\mu}(t))/(C(t)-R(t))}$ ;
18:   end for
19: end while

```

source will be allocated to i in the future. Values in x are always binary (0 or 1) since they are initialized to 0 (line 4) and updated to 1 only (line 14). Constraint (7c) therefore will not be violated.

Let us examine the second constraint (7b). Suppose that the solution is feasible so far. Let $\tilde{j} \in B_i$ be the first bid that breaks the feasibility when added to the current solution, say, in iteration τ . That is, $\exists t$, such that $\sum_{i,j \in \Gamma} d_{i,j}(t) \leq C(t)$ and $d_{\tilde{j},j}(t) + \sum_{i,j \in \Gamma} d_{i,j}(t) > C(t)$, where Γ is the set of bids added to the solution before bid \tilde{j} . Since each single bid cannot exceed the capacity constraint, *i.e.*, $C(t) > R(t) \geq \max_{i,j} d_{i,j}(t)$, we have

$$\sum_{i,j \in \Gamma} d_{i,j}(t) > C(t) - R(t) \Rightarrow \sum_{i,j \in \Gamma} d_{i,j}(t)/(C(t) - R(t)) > 1$$

and that leads to:

$$C(t)\psi_t^{\tau-1} = (T \exp(\theta-1))^{\sum_{i,j \in \Gamma} d_{i,j}(t)/(C(t)-R(t))} > T \exp(\theta-1)$$

which satisfies the first stopping condition in line 14. This implies that iteration $\tau - 1$ is the last iteration, and the bid \tilde{j} would not be added to the solution at all. \square

Dual feasibility: dual fitting. The primal solution is always feasible during the execution, but the dual is not necessarily so. The following lemma ensures that the dual variables can be made feasible through posterior scaling by a carefully chosen factor (dual fitting).

LEMMA 1. *If $(\phi^{\tau-1}, \psi^{\tau-1})$ is the (possibly infeasible) dual solution at the beginning of the τ -th iteration, then $(\phi^{\tau-1}, \epsilon f(\psi^{\tau-1}, j_i^\tau)\psi^{\tau-1})$ is a feasible solution to the dual (10), where $f(\psi, j_i) \triangleq b_{i,j_i}/(\sum_t d_{i,j_i}(t)\psi_t)$, $\epsilon \triangleq \max_{j',j'' \in B_i,i,t} d_{i,j'}(t)/d_{i,j''}(t)$.*

PROOF. According to line 11, we have $\forall i, j, b_{i,j} \leq b_{i,j_i}$. Because ϕ_μ is set to b_{μ,j_μ} where $b_{\mu,j_\mu} \geq b_{\mu,j}$, $\forall \mu \in \mathcal{U}, j \in B_\mu$. That is:

$$\phi_\mu \geq b_{\mu,j}, \forall \mu \in \mathcal{U}, j \in B_\mu$$

which implies that constraint (10a) is satisfied $\forall \mu \in \mathcal{U}, j \in B_\mu$.

Next we examine the bidders $\mu \in \mathcal{M} \setminus \mathcal{U}$. Also note that j_i^τ is decided by a maximization in line 13. Therefore,

$$f(\psi^{\tau-1}, j_i^\tau) \geq \frac{b_{i,j_i}}{\sum_t d_{i,j_i}(t)\psi_t^{\tau-1}}, \quad \forall i \in \mathcal{M} \setminus \mathcal{U} \Leftrightarrow$$

$$f(\psi^{\tau-1}, j_i^\tau) \sum_t d_{i,j_i}(t)\psi_t^{\tau-1} \geq b_{i,j_i}, \quad \forall i \in \mathcal{M} \setminus \mathcal{U} \quad (11)$$

Note that $\epsilon d_{i,j_1}(t) \geq d_{i,j_2}(t), \forall j_1, j_2 \in B_i, i, t$, Eqn. (11) further suggests that the constraint (10a) is respected by the scaled solution, *i.e.*,

$$\epsilon f(\psi^{\tau-1}, j_i^\tau) \sum_t d_{i,j}(t)\psi_t^{\tau-1} \geq b_{i,j_i} \geq b_{i,j}, \forall i \in \mathcal{M} \setminus \mathcal{U}, j \in B_i.$$

This completes the proof. \square

Approximation ratio.

THEOREM 5. *Algorithm 3 computes an a -approximate solution to WDP2 in polynomial-time, and also verifies that the integrality gap of WDP2 is bounded by a , where $a = 1 + \epsilon\Lambda(\epsilon T^{\Lambda-1} - 1)$, $\Lambda = \frac{\theta}{\theta-1}$.*

PROOF. First we analyze the approximation ratio of Algorithm 3. Let $d_1(\tau) = \sum_{i \in \mathcal{M}} \phi_i^\tau$, $d_2(\tau) = \sum_t C(t)\psi_t^\tau$. Let d be the optimal value to the dual (10). Let j_μ denote the bid selected in the τ -th iteration. ω is denoted the last iteration of the loop.

Case 1: Algorithm 3 stops at ω -th iteration where $\mathcal{U} = \mathcal{M}$ and $\sum_t C(t)\psi_t < T \exp(\theta - 1)$. We know that each microgrid wins one bid. In fact the algorithm produces an optimal solution to IP (7) in this case. Theorem 4 guarantees that p_ω is the value of a feasible solution to IP (7). Meanwhile since $\phi_\mu^\omega = \max_{j \in B_\mu} \{b_{\mu,j}\} \geq b_{\mu,j}$, $\forall \mu \in \mathcal{U}, j \in B_\mu$, thus constraint (10a) is satisfied regardless of ψ , *e.g.*, $(\phi^\omega, \psi = 0)$ is a feasible solution, whose value is exactly p_ω as well, to the dual of the LPR. By weak duality for the LP relaxation, any feasible solution to the dual (10) is an upper bound of IP (7). Thus p_ω is the optimal value to IP (7).

Case 2: Algorithm 3 stops at ω -th iteration where

$$d_2(\omega) = \sum_t (C(t)\psi_t^\omega) \geq T \exp(\theta - 1).$$

We analyze the approximation ratio in following two sub-cases.

Sub Case 2.1: If \exists an iteration $\tau \leq \omega$, such that $a \geq \frac{d}{d_1(\tau-1)}$, then an a -approximate ratio is found, since (i) $d_1(\tau)$ is a non-decreasing function of τ because it becomes larger when the iteration continues; (ii) $d_1(\tau - 1) = p_{\tau-1}$, which is the value of the primal solution;

Sub Case 2.2: $a < \frac{d}{d_1(\tau-1)}$, for all iterations $\tau \leq \omega$. For any iteration $\tau \geq 1$, we have:

$$d_2(\tau) = \sum_t C(t)\psi_t^{\tau-1} (T \exp(\theta - 1))^{(d_{\mu,j_\mu}(t))/(C(t)-R(t))}$$

$$= \sum_t C(t)\psi_t^{\tau-1} \left(1 + \frac{\delta}{\frac{C(t)}{R(t)} - 1}\right)^{(d_{\mu,j_\mu}(t)/R(t))}$$

$$\leq \sum_t C(t)\psi_t^{\tau-1} \left(1 + \frac{\delta}{\frac{C(t)}{R(t)} - 1} d_{\mu,j_\mu}(t)/R(t)\right)$$

$$\leq d_2(\tau - 1) + \Delta \sum_t d_{\mu,j_\mu}(t)\psi_t^{\tau-1}$$

where $\delta = \left(\frac{C(t)}{R(t)} - 1\right) \left((T \exp(\theta - 1))^{1/(\frac{C(t)}{R(t)} - 1)} - 1\right)$, $\Delta = \max_t \frac{\delta C(t)}{C(t)-R(t)}$. The first inequality is due to $(1+a)^x \leq 1+ax, \forall x \in [0, 1]$.

Note that $\frac{\delta C(t)}{C(t)-R(t)}$ is a non-increasing function of $\frac{C(t)}{R(t)} > 1$, and $\theta = \min_t C(t)/R(t)$, then $\frac{\delta C(t)}{C(t)-R(t)}$ reaches the maximum when $\frac{C(t)}{R(t)} = \theta$, *i.e.*, $\Delta = \theta(eT^{1/(\theta-1)} - 1)$.

Recall the definition of $f(\psi^{\tau-1}, j_\mu)$, then we have:

$$\sum_t d_{\mu, j_\mu}(t) \psi_t^{\tau-1} = b_{\mu, j_\mu} / f(\psi^{\tau-1}, j_\mu)$$

Since p_τ is the value of the primal solution at the end of τ -th iteration, then $p_\tau - p_{\tau-1} = b_{\mu, j_\mu}$, this leads to:

$$d_2(\tau) \leq d_2(\tau-1) + \Delta \frac{p_\tau - p_{\tau-1}}{f(\psi^{\tau-1}, j_\mu)} \quad (12)$$

Following Lemma 1, we covert the dual variables $(\phi^{\tau-1}, \psi^{\tau-1})$ at the τ -th iteration to $(\phi^{\tau-1}, f(\psi^{\tau-1}, j_\mu) \psi^{\tau-1})$, which is a feasible solution to the dual (10). Therefore we have the following inequality to associate d with d_1 and d_2 :

$$\begin{aligned} d &\leq d_1(\tau-1) + \epsilon f(\psi^{\tau-1}, j_\mu) d_2(\tau-1) \\ \Rightarrow f(\psi^{\tau-1}, j_\mu) &\geq \frac{1}{\epsilon} \frac{d - d_1(\tau-1)}{d_2(\tau-1)} \end{aligned}$$

Recall that for all iterations $\tau \leq \omega$, $a < \frac{d}{d_1(\tau-1)}$, implying:

$$\frac{1}{f(\psi^{\tau-1}, j_\mu)} \leq \epsilon \frac{d_2(\tau-1)}{d - d_1(\tau-1)} \leq \epsilon \frac{a}{a-1} \frac{d_2(\tau-1)}{d}$$

Substitute this bound on $1/f(\psi^{\tau-1}, j_\mu)$ in Eqn. (12):

$$\begin{aligned} d_2(\omega) &\leq d_2(\omega-1) (1 + \epsilon \frac{a\Delta}{(a-1)d} (p_\omega - p_{\omega-1})) \\ &\leq d_2(\omega-1) \exp(\epsilon \frac{a\Delta}{(a-1)d} (p_\omega - p_{\omega-1})) \\ &\leq d_2(0) \exp(\epsilon \frac{a\Delta}{(a-1)d} p_\omega) \end{aligned}$$

the second inequality is due to $1 + x \leq e^x, \forall x \geq 0$.

Note that the stopping condition in this sub case is $d_2(\omega) \geq T \exp(\theta - 1)$ and $d_2(0) = T$, as a result, we have:

$$\begin{aligned} T \exp(\theta - 1) &\leq T \exp(\epsilon \frac{a\Delta}{(a-1)d} p_\omega) \\ \Leftrightarrow d/p_\omega &\leq \epsilon \frac{a\Delta}{(a-1)(\theta - 1)} \end{aligned}$$

Due to the weak duality theorem in linear programming and the relaxation of IP (7), the following inequality holds:

$$OPT_{WDP2} / p_\omega \leq d/p_\omega$$

where OPT_{WDP2} is the value of the optimal solution to WDP2. This means d/p_ω is an upper bound of the approximation ratio.

We hence obtain the approximation ratio:

$$a = 1 + \epsilon \Lambda (eT^{\Lambda-1} - 1).$$

Recall that the integrality gap of WDP2 is defined as the ratio between OPT_{LPR2} and OPT_{WDP2} , where OPT_{LPR2} is the value of the objective function for the optimal fraction solution χ^* . We notice that $d \geq OPT_{LPR2}$ and $OPT_{WDP2} \geq p_\omega$, which lead to

$$OPT_{LPR2} / OPT_{WDP2} \leq OPT_{LPR2} / p_\omega \leq d/p_\omega = a$$

Thus Algorithm 3 verifies the integrality gap, providing an integer solution of value at least $1/a$ times OPT_{LPR2} .

Finally we examine the computation complexity of Algorithm 3. The termination conditions ensure that the `while` loop in Algorithm 3 iterates at most $|\mathcal{M}|$ times, linear to the input size. Within

the loop, lines 10-18 can be finished in polynomial time. Therefore, Algorithm 3 runs in polynomial time overall. \square

Exploiting the dual fitting result in Lemma 1 and LP duality, we prove that Algorithm 3 guarantees an a -approximation of social welfare, as well as verifying an integrality gap of WDP2 of a , where $a = 1 + \epsilon \Lambda (eT^{\Lambda-1} - 1)$, $\Lambda = \frac{\theta}{\theta-1}$. In practice, the volume of a grid's power capacity is substantially larger than a single microgrid's demand, *i.e.*, $\theta \gg 1$. The number of predicted time slots T is a relatively small constant. Consequently,

$$\lim_{\theta \rightarrow \infty} a = \lim_{\Lambda \rightarrow 1} (1 + \epsilon \Lambda (eT^{\Lambda-1} - 1)) = 1 + \epsilon(e - 1)$$

which suggests that the approximation ratio a is close to $1 + \epsilon(e - 1)$. When each microgrid only submits one bid, *i.e.*, $\epsilon = 1$, then $a \approx 2.72$.

5. PERFORMANCE EVALUATION

5.1 The microgrid-to-grid market

We adopt the hourly demand curve of the grid in Ontario, Canada from 00:00, Jan. 3, 2013 to 23:59, Jan. 3, 2013 [3] in evaluating the microgrid-to-grid auction. The order of magnitude has been scaled down by 100, under the rationale that microgrids are not expected to fully support the grid, but will act as a supplement supplier. The trend of hourly demand remains the same after such scaling. The maximum and minimum outputs of each microgrid are $P_{max} = 10MW$ and $P_{min} = 0$, respectively. The ramping rate is $\delta = 1MW/h$. Once a generator is turned ON, it must remain active for at least $T_{on} = T_{off} = 2h$. All generators are OFF at the beginning of the simulation.

Performance of the approximation algorithm. We first implement and evaluate Algorithm 2. The results are shown in Fig. 4. Compared with the optimum, the approximation algorithm achieves an impressive performance, approaching the optimum rather closely in most cases. We observe that the cost goes down when the number of microgrids increases. This is because a small number of microgrids means little cheap energy to the grid. In order to satisfy the power demand, the grid has to resort to more expensive alternatives.

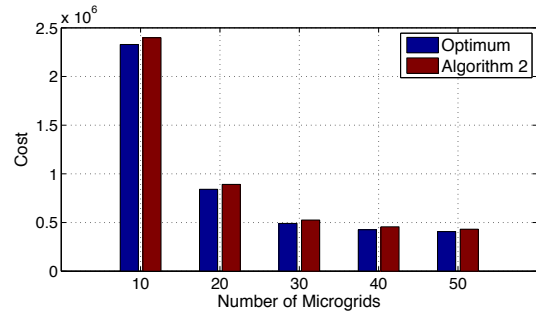


Figure 4: A comparison between optimum and Algorithm 2.

We also compare the energy purchased from plants to supplement the temporary deficits in energy, as illustrated in Fig. 5. We observe that the grid purchases a large amount of energy from plants in the first few hours. This is because all generators are OFF at the beginning. Even if they want to sell energy to the grid, they cannot increase the energy output abruptly due to the UCP3 constraint. In this case, the grid needs to purchase energy from plants or batteries at a relatively high cost. Later a large number of microgrids could help decrease the cost.

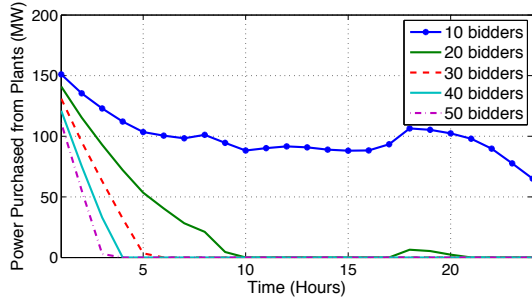


Figure 5: A comparison of energy purchased from plants to supplement the deficits in energy, among different numbers of bidders.

5.2 The grid-to-microgrid market

In this case, microgrids submit bids to purchase energy from the grid for satisfying their own demands. We again use the hourly zonal demands data, shown in Fig. 6, from Ontario, Canada [3] to drive the simulation. These data have been scaled down to meet the reality in practical microgrids, *e.g.*, a community or a university. The trend in demand fluctuation remains the same. We use these data to emulate a number of microgrids, from 10 microgrids to 80 microgrids. The total capacity of the grid is set according to Fig. 3.

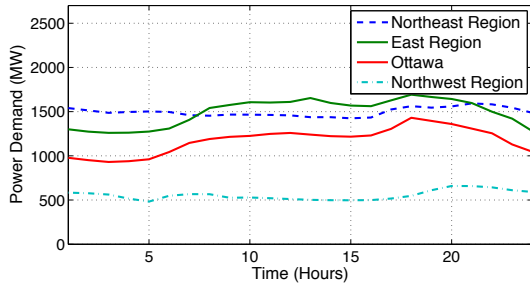


Figure 6: Hourly power demands from 00:00 Jan. 3, 2013 to 23:59 Jan. 3, 2013 in the Northeast, Northwest, East and Ottawa regions in the province of Ontario, Canada [3].

Performance of the Approximation Algorithm. We first investigate the performance of the proposed approximation algorithm, Algorithm 3. The number of microgrids (bidders) varies from 10 to 80. We compare the social welfare of Algorithm 3 with the optimum as well as the theoretical bounds, as illustrated in Fig. 7. We observe that the performance of Algorithm 3 in terms of social welfare, is close to the optimum, and is much better than suggested by the worst-case theoretical guarantee we have been able to prove.

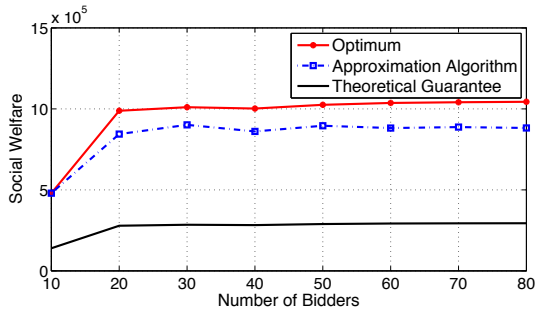


Figure 7: A comparison among the optimum, Algorithm 3 and theoretical bounds.

We further study the performance of the proposed approximation algorithm when demand predictions in $d_{i,j}$ are not perfect, when microgrids underestimate their near future demand by a percentage within 5% and 40%, as shown in Fig. 8. Their bidding prices for unit cost (\$ per MWh) remain the same. We observe a close to linear relationship between the achieved social welfare and the prediction error, and the social welfare decreases when the error grows.

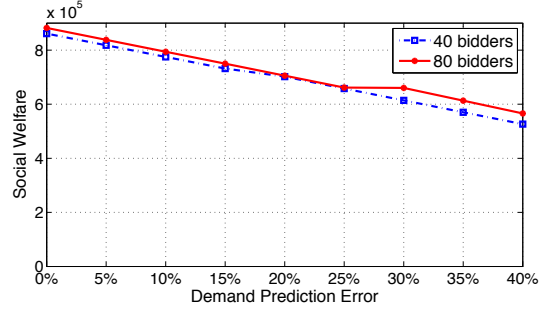


Figure 8: Social welfare achieved by Algorithm 3 under demand prediction errors, when the numbers of bidders are 40 and 80, respectively.

The Randomized Auction. By applying the ellipsoid method and the approximation algorithm, we implement the randomized auction. We obtain the average social welfare by simulating each auction for 20 times, as shown in Fig. 9. The results show that the expected social welfare fluctuates around the theoretical bound. Fig. 10 shows the corresponding total payments. Note that the seemingly high social welfare achieved by the fractional VCG auction is actually infeasible, since the fractional solution to WDP2 cannot be implemented in practice.

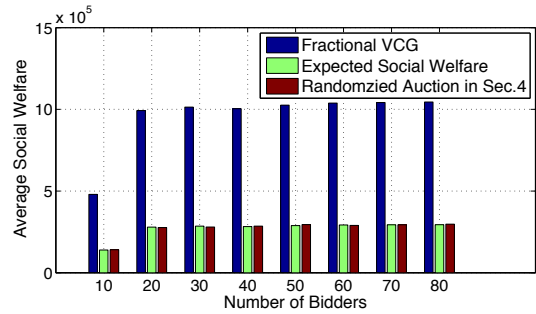


Figure 9: Social welfare of the randomized auction, compared with the fractional VCG auction.

6. CONCLUSION

A fundamental trend in power grid evolution is the proliferation of distributed generation through the microgrid paradigm. The integration of microgrids with traditional power grids requires solutions to both technical and economic challenges. This work is among the first that addresses the latter, through the design of computationally efficient electricity auctions that target good social welfare and truthfulness. The auction designed for the microgrid-to-grid market further represents the first electricity auction mechanism that explicitly models the UCP constraints that are critical in power grid optimization. The proposed auction mechanisms are not completely ready for immediate application in practice, due to its

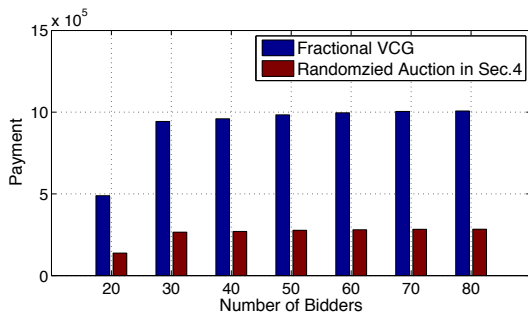


Figure 10: Total payment of the randomized auction, compared with the fractional VCG auction.

limitations in assumptions and computational overhead. We hope the gap will be closed by future work.

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8. REFERENCES

- [1] *Distributed generation*, Wikipedia, http://en.wikipedia.org/wiki/Distributed_generation.
- [2] *ENMAX Energy Rates*, <https://www.enmax.com/Energy/Res/My+Account/Current+EasyMax+Rates.htm>.
- [3] *Independent Electricity System Operator (IESO)*, <https://www.ieso.ca/>.
- [4] *Microgrid Market - 2013 Report*, <http://www.stockhouse.com/News/Press-Releases/2013/09/26/Microgrid-Market-2013-Report>.
- [5] K. Aoki, M. Itoh, T. Satoh, K. Nara, and M. Kanezashi. Optimal Long-Term Unit Commitment in Large Scale Systems Including Fuel Constrained Thermal and Pumped-Storage Hydro. *IEEE Transactions on Power Systems*, 4(3):1065–1073, 1989.
- [6] M. Barnes, J. Kondoh, H. Asano, J. Oyarzabal, G. Ventakaramanan, R. Lasseter, N. Hatziaargyriou, and T. Green. Real-World MicroGrids-An Overview. In *Proc. of IEEE SoSE*, 2007.
- [7] P. Briest, P. Krysta, and B. Vöcking. Approximation Techniques for Utilitarian Mechanism Design. In *Proc. of ACM STOC*, May 2005.
- [8] R. Carr and S. Vempala. Randomized Metarounding. *Random Struct. Algorithms*, 20(3):343–352, May 2002.
- [9] E. Clarke. Multipart Pricing of Public Goods. *Public Choice*, 11(1):17–33, 1971.
- [10] A. Dimeas and N. Hatziaargyriou. Operation of a Multiagent System for Microgrid Control. *IEEE Transactions on Power Systems*, 20(3):1447–1455, 2005.
- [11] S. Dughmi and T. Roughgarden. Black-Box Randomized Reductions in Algorithmic Mechanism Design. *SIAM Journal on Computing*, 43(1):312–336, 2014.
- [12] B. Dunn, H. Kamath, and J. Tarascon. Electrical Energy Storage for the Grid: A Battery of Choices. *Science*, 334(6058):928–935, 2011.
- [13] A. Frangioni, C. Gentile, and F. Lacalandra. Solving Unit Commitment Problems with General Ramp Constraints. *International Journal of Electrical Power & Energy Systems*, 30(5):316–326, 2008.
- [14] M. Grötschel, L. Lovász, and A. Schrijver. *Geometric Algorithms and Combinatorial Optimization*. Springer-Verlag, 1988.
- [15] T. Groves. Incentives in Teams. *Econometrica: Journal of the Econometric Society*, pages 617–631, 1973.
- [16] X. Guan, Q. Zhai, and A. Papalexopoulos. Optimization Based Methods for Unit Commitment: Lagrangian Relaxation Versus General Mixed Integer Programming. In *Proc. of IEEE PES General Meeting*, 2003.
- [17] A. Hawkes and M. Leach. Modelling High Level System Design and Unit Commitment for a Microgrid. *Applied Energy*, 86(7-8):1253 – 1265, 2009.
- [18] H. Kellerer, U. Pferschy, and D. Pisinger. *Knapsack Problems*. Springer, 2004.
- [19] R. Lasseter and P. Paigi. Microgrid: A Conceptual Solution. In *Proc. of IEEE PESC*, 2004.
- [20] R. Lavi and C. Swamy. Truthful and Near-Optimal Mechanism Design via Linear Programming. In *Proc. of IEEE FOCS*, Oct. 2005.
- [21] X. Li, S. Y.J., and H. S.B. Study on Power Quality Control in Multiple Renewable Energy Hybrid MicroGrid System. In *Proc. of IEEE Power Tech*, July 2007.
- [22] Z. Li, B. Li, and Y. Zhu. Designing Truthful Spectrum Auctions for Multi-hop Secondary Networks. *IEEE Transactions on Mobile Computing*, to appear.
- [23] L. Lu, J. Tu, C.-K. Chau, M. Chen, and X. Lin. Online Energy Generation Scheduling for Microgrids with Intermittent Energy Sources and Co-generation. In *Proc. of ACM SIGMETRICS*, 2013.
- [24] B. McGuire. Power Auctions and Intertemporal Production Costs: Dealing with Unit Commitment. Technical report, 1997.
- [25] H. Minooei and C. Swamy. Truthful Mechanism Design for Multidimensional Covering Problems. In *Proc. of ACM WINE*, 2012.
- [26] A. Mu’Alem and N. Nisan. Truthful Approximation Mechanisms for Restricted Combinatorial Auctions. *Games and Economic Behavior*, 64(2):612–631, 2008.
- [27] J. Nicolaisen, V. Petrov, and L. Tesfatsion. Market Power and Efficiency in a Computational Electricity Market with Discriminatory Double-Auction Pricing. *IEEE Transactions on Evolutionary Computation*, 5(5):504–523, 2001.
- [28] N. Padhy. Unit Commitment - A Bibliographical Survey. *IEEE Transactions on Power Systems*, 19(2):1196–1205, 2004.
- [29] L. Tesfatsion. Auction Basics for Wholesale Power Markets: Objectives and Pricing Rules. In *Proc. IEEE PES General Meeting*, 2009.
- [30] A. Tsikalakis and N. Hatziaargyriou. Centralized Control for Optimizing Microgrids Operation. *IEEE Transactions on Energy Conversion*, 23(1):241–248, 2008.
- [31] W. Vickrey. Counterspeculation, Auctions, and Competitive Sealed Tenders. *The Journal of finance*, 16(1):8–37, 1961.
- [32] L. Zhang, Z. Li, and C. Wu. Dynamic Resource Provisioning in Cloud Computing: A Randomized Auction Approach. In *Proc. of IEEE INFOCOM*, April 2014.